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Stochastic modeling of the optimal management of an autonomous microgrid

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Abstract

In non-interconnected areas, the efficient use of renewable energies requires optimal management of electricity consumption. The site studied is the "Cirque de Mafate" on Reunion Island. Our laboratory has developed a mixed integer linear programming model which minimizes the electricity consumption of a cluster of houses. This model is deterministic. Our study focuses on the stochastic part, it aims to model, optimize and simulate the stochastic operation of an autonomous microgrid by mutualizing production and storage resources. A study for the solar resource forecasting is performed, using nonparametric methods for the estimation of probability density functions. Indeed, the prediction of the intermittent resource and the combination of production sources are the keys to the good functioning of a microgrid in autonomous mode. One of the strategies found is to aim for auto-consumption for three days if the solar forecast is pessimistic, a part of the energy is then reserved at the battery level for the next two days. The results allow to evaluate the performance of the system in front of random constraints and to make decisions.

Keywords: Mixed integer linear programming, modeling physical systems, nonlinear optimization under constraints, smart grid

1. Introduction

In isolated sites, decentralized electrification presents the most economical solution for the comfort of the inhabitants. However, the implementation of a cluster of houses in an electrical micro-grid requires optimal management to achieve user self-consumption. To set up the model, we describe the different types of individual consumption and the local energy production available. Power management leads to a large integer mixed linear programming system. Our study focuses on three houses in Roche Plate in the cirque of Mafate of Reunion Island. This paper presents the stochastic experimentation for one house. In the cirque of Mafate, there is no road. All access, including for supplies and emergencies, is on foot or by helicopter. There is no main power supply. The inhabitants produce their own electricity thanks to solar panels, with battery storage, and diesel generators as back-up. However, fuel for the latter must be brought in by helicopter at a high cost [1].

For the implementation of the autonomous microgrid, it is necessary to develop a model that minimizes the energy consumption without degrading the satisfaction of the users [2], [3]. This involves developing an optimal energy management tool that is a large mixed linear program. This model is constrained by physical criteria and the users' wishes. The developed model is deterministic, but the insufficiency of the deterministic model leads us to stochastic models whose goal is to take into account the uncertainties of the physical parameters of the system and the parameters of users. Indeed, the intermittency of the solar resource is taken into account in this paper. The novelty of this work is that we investigate a stochastic model for an optimal energy management. The objective is to model, optimize and simulate the stochastic operation of autonomous micro-grids by mutualizing production and storage resources.

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2. Problem Statement

Our study site is on Roche Plate, in the «Cirque de Mafate» of Reunion Island. Three neighboring houses are concerned. Two of them are inhabited by families with 1 or 2 children, the third is a lodging offering 3 rooms to accommodate possible hikers (tourists) [1]. For this article, we experiment for one house. The studied house is composed of electrical appliances such as refrigerators, freezers, lamps, television, portable radio, HIFI, washing machine [1].

Some electrical appliances will be "defferable" i.e. the energy demand can be shifted throughout the day until a much more suitable time slot is found for their execution. The production device consists of photovoltaic panels (327W/unit) [1].

3. Mathematical Models

3.1. Deterministic approach

At the source level, the deterministic approach consists in finding a clear sky model to estimate the solar radiation of a day. The theoretical model designed to estimate the solar radiation of a day is the Bird model [1], [4].

The description of each electrical consumption is given during a period of T hours [1], [3]. Typically $T = 24$ hours for one day. For this period, the time interval $[k, k+1]$, is defined for $k \in \{0, \ldots, T-1\}$ in the cluster of three houses. The problem is formulated like an optimization problem with constraints [1], [2], [3] with a formula for the objective function to be minimized. The corresponding list of parameters is defined by the following notations [1], [5]. If $f_j(i)$ denotes the end of the consumption i in the house j, $f_j^{min}(i)$ and $f_j^{max}(i)$ refer to the lower and upper bound of $f_j(i)$, $f_j^{opt}(i)$ is the optimal value corresponding to the resident's wish for the end of the consumption i in the house j. The evaluation of $U_j(i)$, the distance between $f_j(i)$ and $f_j^{opt}(i)$, is proposed.

Starting by:

$$f_j^{min}(i) \leq f_j(i) \leq f_j^{max}(i) \tag{1}$$

Then $U_j(i)$ the distance between $f_j(i)$ and $f_j^{opt}(i)$ is defined by [1]:

$$U_j(i) = \begin{cases} \frac{f_j(i) - f_j^{opt}(i)}{f_j^{max}(i) - f_j^{opt}(i)} & if\ f_j(i) > f_j^{opt}(i) \\ \frac{f_j^{opt}(i) - f_j(i)}{f_j^{opt}(i) - f_j^{min}(i)} & if\ f_j(i) \leq f_j^{opt}(i) \end{cases} \tag{2}$$

Which should be reduced as little as possible in order to satisfy greatly the comfort of the users.

It is easy to verify that $0 \leq U_j(i) \leq 1$, and to understand that the more $U_j(i)$ is close to 0, the more the user is satisfied. The last formula (2) can be written shortly by [1]:

$$U_j(i) = \delta_{j_u}(i) \frac{\left(f_j^{opt}(i) - f_j(i)\right)}{f_j^{opt}(i) - f_j^{min}(i)} + \left(1 - \delta_{j_u}(i)\right) \frac{\left(f_j(i) - f_j^{opt}(i)\right)}{\left(f_j^{max}(i) - f_j^{opt}(i)\right)} \tag{3}$$

where $\delta_{j_u}(i) \in \{0 ; 1\}$ is the function defined by:

$$\delta_{j_u}(i) = 1 \text{ if and only if } f_j(i) \leq f_j^{opt}(i) \tag{4}$$

Finally, equation (3) is written in the form [1]:

$$U_j(i) = \left(\frac{f_j^{opt}(i)}{f_j^{opt}(i)-f_j^{min}(i)} + \frac{f_j^{opt}(i)}{f_j^{max}(i)-f_j^{opt}(i)}\right) \times \delta_{j_u}(i) - \left(\frac{1}{f_j^{opt}(i)-f_j^{min}(i)} + \frac{1}{f_j^{max}(i)-f_j^{opt}(i)}\right) \times z_{j_u}(i) + \frac{\left(f_j(i)-f_j^{opt}(i)\right)}{\left(f_j^{max}(i)-f_j^{opt}(i)\right)} \quad (5)$$

where

$$z_{j_u}(i) = \delta_{j_u}(i) \times f_j(i) \quad (6)$$

The objective function to be minimized is given by:

$$J = \sum_{j=1}^{3} \sum_{i=1}^{I_j} \sum_{k=1}^{T} E_j(i,k) + \sum_{j=1}^{3} \sum_{i=1}^{I_j} U_j(i) \quad (7)$$

Here, I_j denotes the number of electrical appliances in the house n$^\circ j$, for j going from 1 to the number 3 of houses. During the range time $[k\,\Delta t, (k+1)\Delta t]$, $E_j(i,k)$ is the energy (Wh) consumed by the service n$^\circ i$ in the house n$^\circ j$ and $\Delta t = 1$.

Let us now come to specify all the constraints governing the storage of the battery [1], [6].
For each instant t (in hour), $1 \le t \le T$, the balance for the supply power is:

$$-P_{Bin}(t) + P_{Bout}(t) - P_{Load}(t) + P_{PV}(t) \ge 0 \quad (8)$$

where $P_{Bin}(t)$ is the power stored in the battery and $P_{Bout}(t)$ the power supplied by it, $P_{Load}(t)$ is the energy consumed by the electric devices, $P_{PV}(t)$ the energy produced by the photovoltaic panel.

For the linearization of the problem, we follow the procedure described by Bemporad et al [7]. Additional inequality constraints will be added to the system due to the introduction of the new variables by the method used in [3], [8]. The MILP (Mixed Integer Linear Programming) formulation is then solved to obtain the allocation of services throughout the day [1].

The evolution of the battery state of charge $SOC(t)$ is governed by the following equation: for all $1 \le t \le T-1$,

$$SOC(t+1) = SOC(t) + \left(\omega_{B_{in}}(t) - \omega_{B_{out}}(t)\right) \times \Delta(t) \quad (9)$$

where $\omega_{B\_in}(t)$ and $\omega_{B\_out}(t)$ are respectively the battery current of charge and discharge.

The battery state of charge is bounded by the upper limit SOC_{max} and lower limit SOC_{min} : for all $1 \le t \le T$,

$$SOC_{min} \le SOC(t) \le SOC_{max} \quad (10)$$

This constraint means that the battery should not be discharged or charged beyond some limits in order to protect it from damage and to extend its life.

Finally, the battery currents of charge and discharge are bounded by using a control parameter $\alpha(t)$, a logic variable, satisfying: for all $1 \le t \le T$,

$$\begin{cases} 0 \le \omega_{B_{in}}(t) \le \alpha(t) \times w_{max\_c} \\ 0 \le \omega_{B_{out}}(t) \le \left(1-\alpha(t)\right) \times w_{max\_d} \end{cases} \quad (11)$$

where ω_{max_c} and ω_{max_d} are respectively the maximum limit value of the battery current of charge and discharge.

3.2. Stochastic approach

In the stochastic part, a model taking into account uncertainties of the enter parameters is developed.

Due to the intermittence of the solar radiation, deterministic model is not sufficient to predict daily solar radiation. Indeed, to make the prediction of solar radiation much more realistic, we used the actual data from the test site (Roche Plate, Mafate), and estimate its distribution for each hour.

Two nonparametric methods for the estimation of probability density functions were therefore used to estimate the distribution of solar radiation: the histogram method and the kernel method. For the kernel method, we used the Gaussian kernel and the Epanechnikov kernel [9], [10], [11], [12], [13].

Indeed, it is possible to estimate the probability density function from a sample of n observed values of X denoted by $x_1,\ x_2, \dots,\ x_n$; which are assumed to be independently and identically distributed according to the law of X [9]. The aim is to deduce from the sample an estimate of the probability density function of the random variable X.

Let $h \in \mathbb{R}_+^*$ be a parameter called bin width. Let $([kh,(k+1)h))_{k\in\mathbb{N}}$ be a partition of $\mathbb{R}_+$. The histogram method gives the following estimator of the probability density function:

$$\hat{f}_h(x) = \frac{1}{nh}\sum_{k=1}^{+\infty} N_k 1_{[kh,(k+1)h)}(x) \qquad \forall x \in \mathbb{R} \tag{12}$$

where $1_{[kh,(k+1)h)}(.)$ is the indicator function of the interval $[kh,(k+1)h)$ and $N_k = \# \{i: x_i \in [kh,(k+1)h), 1 \le i \le n\}$ is the number of observations in $[kh,(k+1)h)$.

However, the histogram estimator has a non-negligible defect that is to be non-continuous. To obtain a continuous probability density function, we use the kernel (or Parzen) method. This method is a generalization of the histogram method [11]. The probability density function is then estimated by:

$$\hat{f}_h(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right) \qquad \forall x \in \mathbb{R}, \tag{13}$$

where K is an even probability density function called kernel and $h \in \mathbb{R}_+^*$ is a parameter called bandwidth, which governs the degree of smoothness of the estimator.

Now let come to study the distribution of solar radiation for each hour using annual radiation data from the Roche Plate test site, Mafate:

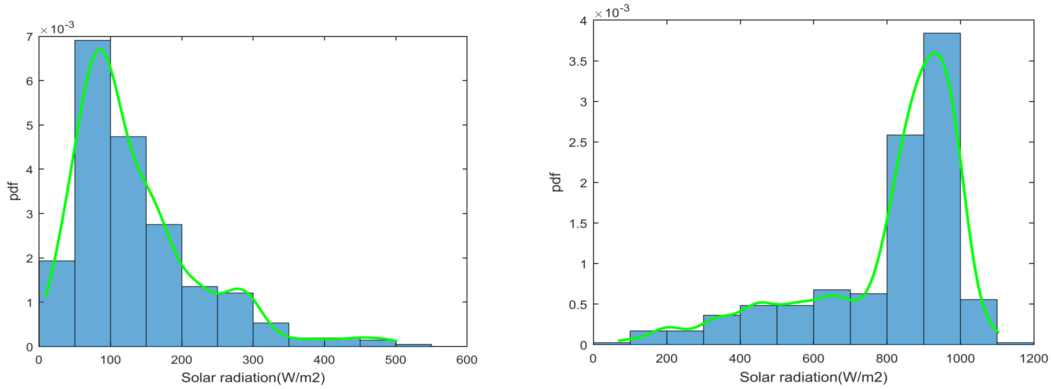
- At 5 a.m., 6 a.m., 7 a.m., 4 p.m., 5p.m, 6p.m, and 7 p.m., the solar radiation may be modeled by random variables following log-normal distributions.
- From $k = 8$ a.m. to $k = 12$ a.m., the solar radiation $\mathcal{R}_k$ may be modeled by:

$$\mathcal{R}_k = M_k - S_k \tag{14}$$

where $M_k > 0$ is the upper bound of the solar radiations given by the data and S_k is a random variable following a log-normal distribution truncated to the interval $[0; M_k]$.

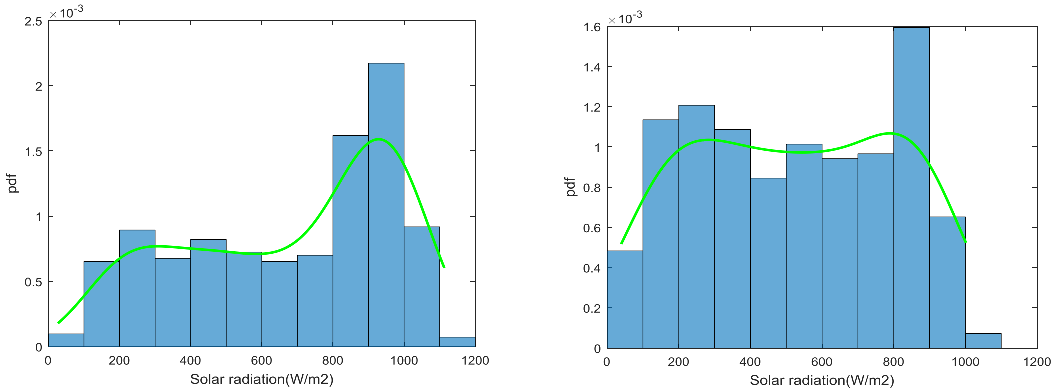
- From 1 p.m. to 3 p.m., the solar radiation may be modeled by random variables following bimodal distributions, their probability density functions are convex combinations of two gaussian densities f_1 and f_2 that is for some $\alpha \in]0\,;1[$:

$$f = \alpha\, f_1 + (1-\alpha)\, f_2 \tag{15}$$



(a) (b)

Fig. 1. (a) Estimated probability density function of solar radiation at 4 p.m., (b) estimated probability density function of solar radiation at 11 a.m



(a) (b)

Fig. 2. (a) Estimated probability density function of solar radiation at 1 p.m., (b) estimated probability density function of solar radiation at 2 p.m.

Figs. 1 and 2 show the plots of probability density functions estimators for solar radiation obtained by nonparametric methods. To illustrate the estimated density obtained, we took the following times: 11 a.m., and 1p.m and 2 p.m. and 4 p.m. We can see that the random variable of the estimated density follows the proposed laws.

Thus, to take into account uncertainties, we are interested in the method of propagation of uncertainties which consists in associating with the input quantities (input parameters) random variables determined by their probability distributions. For the deterministic part, we use the MILP as an optimization tool, of which all the algorithms that can be involved are already detailed by the preceding mathematical formulations. The objective of the stochastic part is to introduce uncertainties into these algorithms using the method of propagation of uncertainty [14], [15]. Thus, the uncertainty of the input parameters impacts the output variables. This modelling allows to explore the possible states of the system and their consequences when the system constraints are not satisfied. So, in the stochastic approach, the energy produced by the photovoltaic panel $\widetilde{P_{PV}}(t)$ is a random variable that depends on the intermittence of the solar radiation. For output variables, $\widetilde{P_{Bin}(t)}$ is the random variable of power stored in the battery, $\widetilde{P_{Bout}(t)}$ the random variable of power supplied by it and $\widetilde{SOC(t)}$ the random variable of the evolution of the battery state of charge.

4. Numerical Simulations of Stochastic Experiment

For the stochastic modelling of the autonomous micro-grid, the intermittency of production is taken into account, thus the photovoltaic production is modelled taking into account the predicted solar radiation, that is obtained by using the experimental data of the studied site, the non-parametric estimation methods and the numerical simulations. Thus, we can predict the possible productions for each day, for each month and for each season and forecast our ability to meet the demand, and therefore decide on the amount of energy to keep at the battery level for the following days. As said before, self-consumption for three days is targeted when the solar forecast becomes pessimistic for the next two days.

The Fig. 3 shows us the consumption of electrical appliances in a house, the electrical consumptions are characterized by permanent services from 1 a.m. hour to 12 p.m. due to the activity of the refrigerators and freezers. The punctual loads are localized around 12 a.m., in the solar radiation zone, except for devices which are only useful in the evening to serve the needs of the night such as lights. Indeed, this configuration corresponds to the optimal distribution of service proposed by the solver. The high demand of loads around 12 a.m., or in the solar radiation zone minimizes the use of the battery.

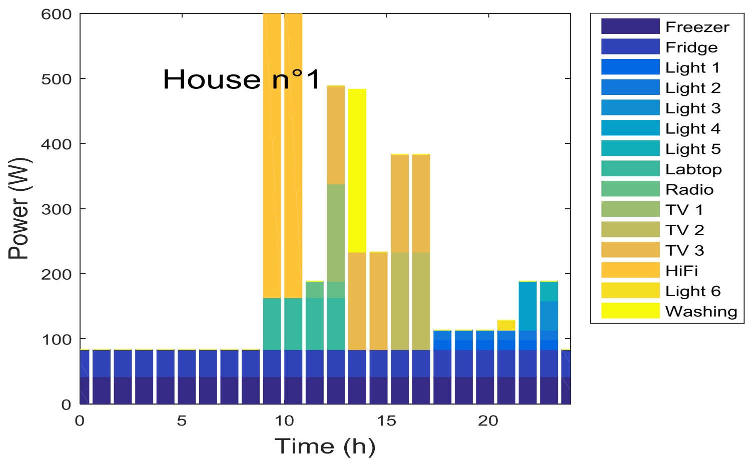


Fig. 3. Consumption of electrical appliances in a house

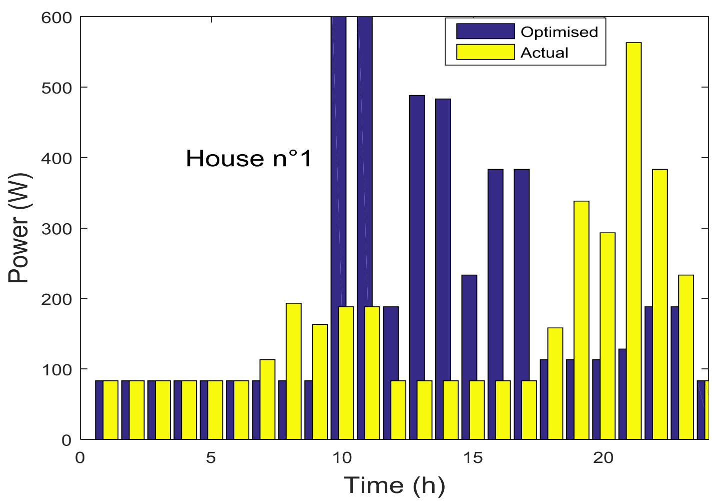


Fig. 4. Classic and optimized daily electrical consumptions for a house

As shown in Fig. 4, a comparison between the current use of electrical devices and the optimized configuration proposed by the solver allows to highlight the low power consumption at the end of the day for the optimized consumption. In fact, in the optimized configuration, only the permanent services and the low-power evening lights operate from 6 p.m.

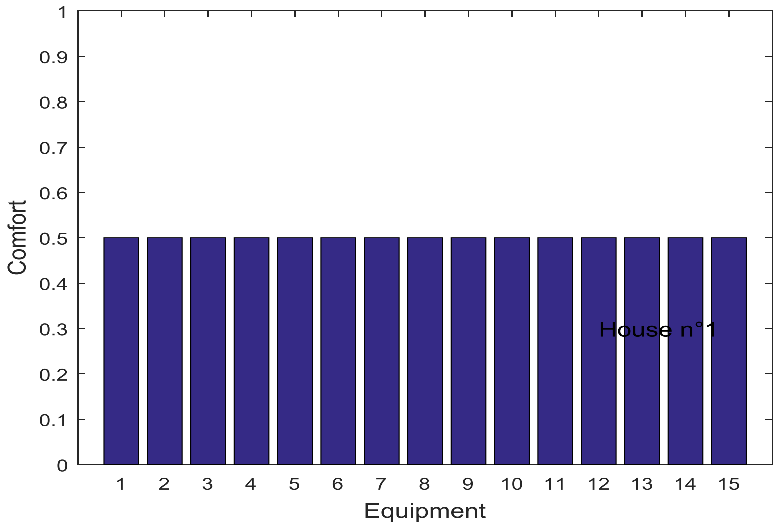
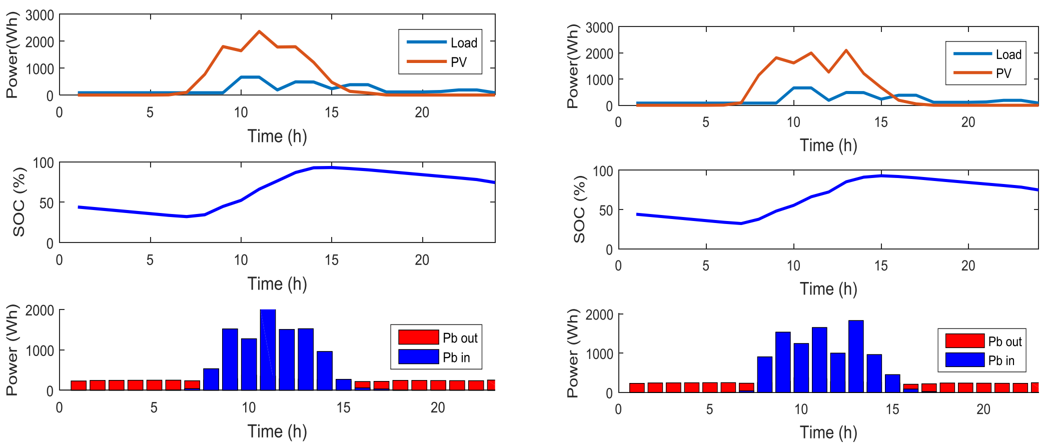


Fig. 5. Evaluation of users' comfort for each consumption service

For the comfort, as shown in Fig. 5, the satisfaction graph gives the difference between the time service calculated by the solver and the desired time giving by the users. A constant comfort level of 50 % is observed, it allows to validate the configuration of the proposed use of the services, the goal is to minimize consumption without too much degrading users 'satisfaction.



(a) (b)

Fig. 6.(a) scenarios 1 of predicted production and optimized consumption, evolution of the battery state of charge and variation of charge and transfer power for the battery, (b) scenarios 2 of predicted production and optimized consumption, evolution of the battery state of charge and variation charge and transfer power for the battery

Fig. 6 show us the two of the possible scenarios of the random variable of the production, the random variable of the evolution of the battery state of charge, the random variable of the power stored in the battery and the random variable of power supplied by it. As shown in the figures, the intermittency of the solar production is observed.

Seeing the two scenarios of the random variable of the evolution of the state of charge of the battery, we can observe in both cases a state of charge higher than 60% at the end of the day. The graph allows us to follow the dynamics of flux exchanged between production and storage. The power $\widetilde{P_{B\iota n}}$ stored in the battery logically follows the solar radiation curve. Therefore, the stochastic study demonstrates the performance of the system in front of intermittent source. The system can adapt well to random constraints.

In the stochastic study, we can have a degraded day for the solar production and for that, the only source of production will be the battery, and it is precisely in this case that we have to minimize the power consumption, and therefore the power supplied by the battery in the precedent clear sky day so that the system is autonomous for the next two days.

Thus, the study of the solar radiation forecast will help us to predict the optimistic or pessimistic day of solar production and to make decisions on the management of the microgrid system.

5. Conclusion

The results show that the solver adapts well to the stochastic operation of the microgrid system. However, efficient management of cluster of house is an alternative to obtain the most efficient use of renewable energy resources. Looking for the minimum energy consumption for one house is not enough to optimize the production system. Modeling of nanogrid at the scale of a house must be use to upscale at the level of microgrid. The main advantage of the production and the participative consumption is to mix almost electrical devices. This combination of services provides greater flexibility to renegotiate deliveries or reduced load shedding. The grouping of houses allows to attain the autonomy of the electrical network [1].

In this study, the intermittence of the resource is taken into account and the prediction of the solar radiation, thus the solar production allows us to make decision and validate the performance of the system in front of the intermittent source. The consideration of uncertainties on power consumption is currently in progress. The next step is to extend the stochastic study of one house to the scale of the three houses, followed by the consideration of demand variation. The study of the system performance will then be carried out followed by decision making, due to the prediction of the resource and the variation of the demand.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

All authors conducted the research and analyzed the data; Paulisimone Rasoavonjy, Oanh Chau and Sylvain Dotti wrote the paper; all authors had approved the final version.

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