Welfare maximization under a three-choice minority game model for energy demand allocation

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Abstract

Microgrids have been increasingly researched as a promising solution for providing energy to off-grid rural communities. Among the challenges of deploying microgrids to these communities is the problem of developing simple demand management schemes so that peaks and troughs in the consumers' load demand can be minimized. A usual demand management scheme involves awarding consumers who utilize energy at times of low demand and penalizing those who utilize energy at times of high demand. Such schemes, however, may not always maximize user welfare. In this work, welfare analysis for a three time-window scheme for a simple microgrid is conducted. The study investigates whether ideal agent distributions that maximize social welfare across these time windows of energy usage can be computed assuming homogeneous agent utility and linear costs. A three-choice minority game was then developed as an allocation scheme for the grid. The game was simulated to determine whether these ideal agent distributions can be achieved within the game. Simulation results show that agent attendances over time approach the ideal agent distributions for some penalization schemes. The developed model therefore has a promising potential to be applied as a demand management scheme.

Keywords: microgrid demand management, minority games, game theory

1. Introduction

Conventionally, demand management schemes have been designed for traditional power grids. A common scheme is to penalize consumers who use energy at peak hours and award consumers who use energy at off-peak hours [1]. This concept is not new, and several time-of-use schemes have already been developed and analyzed [2]. With the rising popularity of microgrids and smart grids, however, there has been a growing interest to revisit the demand management problem [3]. While microgrid setups can vary in size and scope, off-grid rural communities often work well with simple setups. As such, there is a need to develop equally simple demand management schemes for these grids.

This work considers the problem of managing consumer demand across three time-of-use windows for a simple microgrid setup. This work aims to determine whether social welfare can be maximized in this setup. Closed-form expressions for the ideal agent distributions that would maximize social welfare across these time windows are computed. A three-choice minority game model is then developed as a potential allocation scheme. Simulations are then conducted to determine whether the ideal agent distributions that maximize social welfare can be achieved through the model.

The next section discusses related work that are relevant to the study. Section 3 presents the computations on the ideal welfare-maximizing agent distributions. Section 4 describes the minority game mechanism developed. Sections 5 and 6 present the results and conclusions of the work.

2. Related Work

The original minority game was first proposed by Challet and Zhang [4]. In their formulation, an odd number N of agents need to select one out of two choices in a set of available choices $A = \{0,1\}$ at every time step. The game is therefore binary, since only two choices are available to all agents. The winners of the game are then the agents who have selected the choice chosen by the least number of agents.

In selecting what choice to take, all agents have a memory of the past winning minority choices. This memory is denoted by a binary string of length M recording the past winning choices. This history string of past M winning choices is the only information available to all agents. To make a decision, each agent also has a fixed number n of strategies. While strategies can take on many forms, the simplest strategy maps every possible history string to a choice that the agent will take. At the start of the game, each agent is randomly assigned n number of strategies taken from the superset S of all possible strategies.

More than one strategy can be assigned to each agent. In the original model, each agent picks the best performing strategy so far at every round. To determine the best performing strategy, all strategies of an agent are scored using a scoring function $b_{s_{i,h}}(t)$ (which can be defined in a variety of ways [5]) at the end of every round. Every strategy is scored regardless of whether such a strategy was used or not. The best strategy at any given round is therefore the strategy with the current highest score.

The original minority game has since been extended into a multichoice minority game where agents have more than two choices. Several works developing different variants of multichoice minority games have since been developed (e.g. see Refs. [6][7]). One notable work includes a study by Lam and Leung on multichoice minority games for resource allocation [8]. In their study, agents have more than two resources to choose from; however, each resource can only accommodate a limited capacity of agents.

3.   Welfare Computations

This work is purely economical and simply assumes that the maximum load demand of a target community is serviceable by a simple microgrid. Regardless of the microgrid configuration, the microgrid is assumed to supply a fixed amount of energy over three time-windows in a single day. Only three time windows are assumed for now, as three time windows of energy use are often considered in small grid setups in Philippine rural communities. Users in the community have the full freedom to choose at which time window they will utilize energy; however, as the supply is fixed, the microgrid should ideally not be overloaded with too many users opting to consume energy all at the same time window. To simplify the analysis, we likewise assume that users can only choose one time window during which they will consume energy from the grid. We also assume that agents have a homogeneous valuation, wherein they obtain the same utility from selecting a specific choice. Formally, a homogenous valuation set **v** is defined to be:

$$\mathbf{v} = \rho \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}, \ v_1 + v_2 + v_3 = 1 \text{ and } v_1, v_2, v_3 \in \mathbb{R}^+ \tag{1}$$

where ρ is a scalar and serves merely as a scaling parameter. We similarly choose the cost $C(\xi_j(t))$ incurred by an agent to be linear:

$$C(\xi_j(t)) = \xi_j(t) - \frac{1}{3} \tag{2}$$

where $\xi_j(t)$ is the percentage of agents who have chosen a choice j at time t. Equation 2 is the difference between the percentage of agents at their selected choice and the ideal mean percentage of agents utilizing energy per choice of time window. This ideal mean is set to be one-third of the total agents. Thus, if too many agents are utilizing energy at a time window, they are penalized with greater costs. From Equations 1 and 2, the individual agent welfare $\omega_j(t)$ is therefore the difference of the utility and the cost:

$$\omega_j(t) = \rho v_j - C(\xi_j(t)) \tag{3}$$

The total social welfare is then the sum of the individual welfare values across all agents:

$$\chi(t) = N \sum_{j=1}^{3} \xi_j(t)\, \omega_j(t) \tag{4}$$

The ideal distribution is the percentage of agents per choice that would maximize Equation 4, given Equations 2 and 3 for the utility gained and cost incurred by each agent. For clarity, we denote the ideal distribution using ξ_1^*, ξ_2^*, and ξ_3^*, as the ideal percentage for the first, second, and third choice, respectively. The problem of maximizing social welfare can therefore be written as:

$$\text{argmax}_{\xi_1,\xi_2,\xi_3}\, \chi \qquad \text{subject to: } \xi_1 + \xi_2 + \xi_3 = 1 \tag{5}$$

Since Equation 5 is a simple maximization problem, the straightforward method of using Lagrange multipliers can be used to solve for the closed-form expressions of ξ_1^*, ξ_2^*, and ξ_3^* to obtain these solutions:

$$\xi_1^* = \frac{1}{3} + \rho\left(\frac{v_1}{3} - \frac{v_2}{6} - \frac{v_3}{6}\right) \tag{6}$$

$$\xi_2^* = \frac{1}{3} + \rho\left(\frac{-v_1}{6} + \frac{v_2}{3} - \frac{v_3}{6}\right) \tag{7}$$

$$\xi_3^* = \frac{1}{3} + \rho\left(\frac{-v_1}{6} - \frac{v_2}{6} + \frac{v_3}{3}\right) \tag{8}$$

4. Three-choice Minority Game Model

This study now examines whether consumers in the grid can be influenced to select time windows such that the ideal agent percentages defined in Equations 6-8 can be achieved. For this work, we propose a three-choice minority game scheme. In the proposed model, agents have three possible choices to choose from, i.e. *A = {0,1,2}*. The minority game's mechanics are retained, wherein only one minority group is declared the winner in a round of the game. However, since the model aims for achieving the ideal group size of agents per choice, simply declaring the smallest group of agents to be the winner will not necessarily hold. This is because the smallest group size may not be the ideal group size for that choice. We therefore adopt a win condition similar to Lam and Leung's work [8], where the minority winners are the group of agents whose size does not exceed some desired capacity. The winning group is the group *j* such that:

$$\min |\xi_j^* - \xi_j(t)| \tag{9}$$

With respect to the agent's strategies, the original form of the strategies as string mappings is retained. Again, however, as the model aims to achieve the ideal agent distributions across the choices, the scoring scheme of the agents' strategies needs to consider the difference between the actual group size and the ideal group size. Hence, for an action *j* chosen by the agent at time *t*, the scoring scheme is:

$$b_{s_{i,h}}(t) = \begin{cases} \xi_j^* - \xi_j(t), & \text{if } \xi_j(t) \le \xi_j^* \\ \xi_j^* - [\xi_j(t) + C], & \text{if } \xi_j(t) > \xi_j^* \end{cases} \qquad \textit{C is a fixed constant} \tag{10}$$
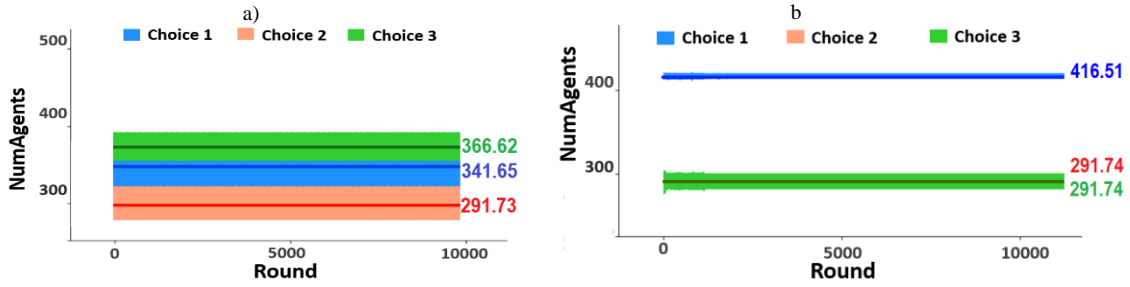
Fig. 1. Number of agents per choice in an average game run. a) valuation set 1 b) valuation set 2

5. Results and Discussion

Thirty-two simulation runs each with 10,000 time steps of the proposed three-choice minority game were performed for $M=2$ with $N=1,000$. For comparison, two homogeneous valuation sets were used: $v_1 = 0.35$, $v_2 = 0.25$, $v_3 = 0.40$, and $v_1 = 0.50$, $v_2 = 0.25$, $v_3 = 0.25$. The resulting ideal agent distributions computed using Equations 6-8 with $\rho= 1$ are $\xi_1^*=34.2\%$, $\xi_2^*=29.2\%$, $\xi_3^*=36.6\%$ for valuation set 1, and $\xi_1^*=41.6\%$, $\xi_2^*=29.2\%$, $\xi_3^*=29.2\%$ for valuation set 2. Lastly, the constant C in Equation 10 was varied with $C=0$, $C=0.001$, $C=0.01$, $C=0.1$, and $C=1$.

Table 1. Mean number of agents per choice. a) valuation set 1 b) valuation set 2

C value	a) Valuation set 1			b) Valuation set 2		
	Choice 1 (ideal: 342)	Choice 2 (ideal: 292)	Choice 3 (ideal: 366)	Choice 1 (ideal: 416)	Choice 2 (ideal: 292)	Choice 3 (ideal: 292)
C=0	341.65	291.73	366.62	416.51	291.74	291.75
C=0.001	341.66	291.73	366.61	416.35	291.82	291.82
C=0.01	341.65	291.73	366.62	416.49	291.75	291.75
C=0.1	341.66	291.72	366.62	416.52	291.74	291.74
C=1	341.66	291.69	366.65	416.47	291.76	291.7

Table 1 shows the mean number of agents per choice for both valuation sets. Visually, Figure 1 shows the average game run. It is evident that the mean number of agents per choice approaches the ideal percentage distribution of agents per slot. There seems to be no observable significant difference on the resulting mean number of agents and their difference from the ideal mean as the constant cost C is increased.

However, it is also evident that these results have been obtained based on very nice assumptions. In reality, while homogeneous valuations are observable in small rural grids, this does not hold for more
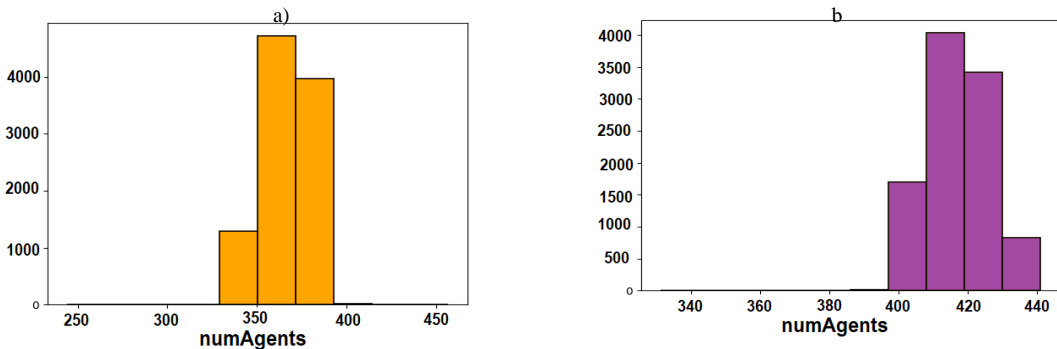

Fig. 2. Agent number histogram in an average game run for C=0. a) valuation set 1 b) valuation set 2

complex grids. Likewise, the proposed model did not consider the total energy margins per sector. While this work has not explicitly considered actual total energy margins, Figure 2 shows the histogram distributions of agent numbers at the choice with the highest valuation for $C=0$ (similar histograms can be obtained for the other C values). These histograms can be used as basis for comparison with the relevant total energy margin for a sector for relevant planning concerns. For instance, Figure 2.b reveals that the highest number of agents in the game hits 440 agents, despite the ideal target of 417 agents. This information can be easily scaled for comparison with the relevant margin of a certain sector for microgrid planning and sizing decisions.

The results are nevertheless encouraging, and many extensions can be done. For future work, we intend to incorporate real demand values of a target rural community in the Philippines for comparison, as well as consider actual data on total energy margins per sector within the Philippine context. In addition, more validation can be done by incorporating more heterogeneity in the valuation sets of the agents. Lastly, considering the actual preferences of agents (as opposed to just the utility values) would also be an avenue for further work, and is the subject of an upcoming study [9].

6. Conclusion

As microgrids have plenty of potential in supplying energy to rural communities, research interest in developing demand management schemes has been renewed. This work considered social welfare maximization in a simple microgrid setup, wherein consumers can use energy from the microgrid at a set time window of their own choosing out of three available time windows of the day. Given a homogeneous utility and a linear cost, closed form expressions were obtained for the ideal agent distribution that would maximize social welfare across the three time windows. A three-choice minority game model was then formulated and simulated to determine if the model can achieve the ideal agent distributions. Results show that the mean numbers of agents per choice have approached the desired agent distribution through the proposed minority game model. Nevertheless, plenty of further work may be done to further improve the model, particularly with respect to direct applications of the model to actual, small microgrid setups and comparisons with actual data sets.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Adrian Roy Valdez conceptualized and formulated the modified three-choice minority game model. Catalina Montes solved the produced the solutions to the simple maximization problem, which Adrian Roy Valdez further verified. Catalina Montes programmed and performed the simulations of the model. Both authors analyzed the data and developed the analysis and conclusions. Both authors have approved the final version of the paper.

Acknowledgements

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