Small signal analysis of dual input buck converter

M M Alam, D C Lu, Y Siwakoti

School of Electrical and Data Engineering, University of Technology Sydney, 2007, Australia

Abstract

This paper presents a small signal modelling and voltage-mode control of a pulse-width modulated (PWM) dual-input DC-DC buck converter. The control of multiple switches in a power converter is the main challenge for multipleinput converters addressed in this paper. Using the concept of linearization and perturbation depicted in circuit averaging technique, the closed-loop small signal model for multi-input DC-DC buck converter is derived. The closed loop control to output voltage transfer function is derived. A brief compensator design is introduced for a multi-input buck converter. In order to control the duty cycles of multiple switches and control the output voltage, a new variable is introduced to relate the duty cycles in the closed loop control to output voltage transfer function. The analysis and controller design are simulated in LTSpice.

Keywords: Small Signal Model, Pulse Width Modulation (PWM), DC/DC Buck Converter

1. Introduction

Nowadays, the power generation sector is inclining towards the use of renewable energy resources in every field such as residential, automotive and aerospace. The main reason behind using renewable resources is their environmental-friendliness, which makes them a better alternative as compared to conventional power generation techniques. However, the performance and reliability of renewable resources is remarkably affected by their intermittent nature [1]. This intermittent nature is badly affecting the complete unification of power generation with renewable resources. Therefore, the optimal solution of this intermittent nature is the combination of renewable resources with various other energy resources [2]. The integration of renewable resources with other energy resources provides the basis of using and analysing power converters. In general, the integration of each energy module into a system requires a dc-dc converter that can result into less efficient, bulky system with more control problems [3]. Therefore, the combination of renewable multiple input power converter having an ability to accommodate more than one inputs of energy resources and provide the output/s according to the user's requirement.

With the help of using the multiple-input converters, the energy can be utilised better and it can also increase the reliability of the overall system [4]. Utilising multiple-input converter can result in different advantages such as less component count, smaller size and control simplicity [5]. The authors in literature [6-8] discussed and analysed several dc-dc multiple-input converters including double-input buck, boost and buck-buck boost converters. A brief comparison study between different converter designs is discussed in [9] where the authors compared various converter topologies on the basis of component count, reliability aspect and the flexibility of the resources. The control objectives of different converter topologies are discussed in [10-14] as it is outlined as the essential part of these topologies. In [10], the authors tried to keep one of the source current as constant with the output voltage regulation in double-input buck-buck boost topology. In [15], the control objective of a double-input buck-boost converter is discussed where the author introduced the concept of power sharing during the load variations of the resources. Finally in [16], the author performed small signal modelling for double-input buck boost

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Corresponding author. E-mail address: mohammadmubashiralam@gmail.com.

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converter but so far, there is no small signal model introduced for double-input dc-dc converter with a simple control strategy. Following the same concept, this paper discussed the small signal modelling and stable control system for a particular circuit topology depicted in Fig. 1.

In this paper, the small signal modelling for multi-input buck converter is discussed in Section 2. In section 3 the steps involved in designing the control of the circuit are explained. In Section 4, the simulation and experimental results will be analysed according to the initial requirements followed by the concluding remarks about selected multiple-input single output topology.

2. Small Signal Modelling of DISO Buck Converter

In order to design a stable control system, this paper analysed the control strategies discussed in [16-19]. In [16], the author proposed a control strategy depending upon independent control of two loops. One control loop was designed for current control while the second loop was controlling the voltage. In this way, the loops were controlling the duty cycles of both the switches independently. One of the limitations of this control strategy was its level of complexity. Another research study in [18] introduced an idea of controlling three switches by a new control strategy. According to the author, a different PWM technique was employed by using two saw tooth waveform carriers. The author used an up-counting sawtooth wave form for one switch while a down-counting saw tooth waveform for the other switch giving a phase difference of 180 degrees [18]. With the help of this technique, the author tried to avoid turning on both switches at the same time without using an external circuit. The two saw tooth waveforms along with the feedback controller providing different duty cycles in order to control the output. The major drawback of this strategy is its application, the strategy is applicable only for the circuit introduced in [18]. While searching for other control strategies, the paper also analysed a simple PWM control strategy introduced in [19]. The author found that circuit is working in two modes i.e. Boost & Buck. Depending upon the boost required for stability, the author used two different negative feedback controllers for the voltage mode control for the circuit. The control strategy in the voltage mode control is controlling one switch by adding both duty cycles and the other switch is controlled by giving a single duty cycle. The limitation of this technique is again its application, as the control strategy was defined more specifically for the circuitry used in [19]. After analysing all the research papers, this paper is going to discuss a new control strategy for a different multi-input buck converter by following the same steps used in [19]. Proceeding the concept of small signal modelling introduced in [16], this paper will apply small signal analysis for a voltage mode control multi-port buck converter using a negative feedback controller. The author in [16] stated all power converters regardless of their topologies, are non-linear systems. Moreover, authors also stated in [16] before using these converters topologies, the systems needs to be linearize by performing small signal modelling. After performing small signal modelling, the feedback controllers can be designed depending upon the linear time invariant models in order to get desired results [20].



Fig. 1. Dual input buck converter

This paper is based on different technical aspects of power converters defined by the authors in [21, 22], in which the authors have established the small signal model analysis for single input dc-dc converters in the literature review. However, there is no small signal modelling analysis carried out for MISO buck converter with a simplest control strategy yet. Therefore, this paper will carry out small signal modelling analysis for MISO buck converter and outline a simplest control strategy. This being the objective, this paper is discussing the small signal modelling of MISO dc-dc buck converter (shown in Fig. 1) in this section.

In order to understand the working of this circuit, one should understand how the circuit works in different operating conditions. The switch S_1 connected with PV source will try to regulate the input voltage. On the other hand, switch S_2 will control the output voltage to user defined pre-set voltage. In this section, the author followed the concept given in [16] in order to calculate the dynamic response of this system using steady-state average equations. The steady-state average equations for selected MISO buck converter are:

$$L\left(\frac{d\hat{t}_{L}}{dt}\right) = \hat{V}_{p}D_{1} + V_{p}\hat{d}_{1} + \hat{V}_{2}D_{2} + V_{2}\hat{d}_{2} - \hat{V}_{c_{2}} + V_{c_{2}}\hat{d}_{2}$$
(1)

$$C\frac{d}{dt}\left(\hat{V}_{c_2} + \frac{\hat{V}_{c_2}}{R}\right) = -i_L\left(\hat{d}_1 + \hat{d}_2\right) + \hat{\iota}_L(1 - D_1 - D_2)$$
(2)

In these equations, D_1 and D_2 are the duty cycles of the switches $S_1 \& S_2$ respectively. By using the fundamental knowledge of converters [22], the author of this paper applied basic step of averaging the voltage across the inductor during one switching transition in order to get the steady-state equation for inductor stated in equation (1). Similarly, the equation (2) is the result of averaging the current across the capacitor. After getting equation (1) and (2), the author followed the concept of perturbing and linearizing the equations. During perturbing, the product of all the small signal perturbed ac terms is neglected. After this step, Laplace transform is applied in order to convert both equations into the frequency domain for critical analysis [21]. The equations are:

$$L S \hat{\imath}_{L} = \hat{V_{p}}(S)D_{1} + V_{p}\hat{d}_{1}(S) + \hat{V}_{2}(S)D_{2} + V_{2}\hat{d}_{2}(S) - \hat{V}_{c_{2}}(S) + V_{c_{2}}\hat{d}_{2}(S)$$
$$L S \hat{\imath}_{L} = \hat{V_{p}}(S)D_{1} + V_{p}\hat{d}_{1}(S) + \hat{V}_{2}(S)D_{2} + (V_{2} + V_{c_{2}})\hat{d}_{2}(S) - \hat{V}_{c_{2}}(S)$$
(3)

$$\widehat{V}_{c_2}(S)\left(CS + \frac{1}{R}\right) = -i_L\left(\widehat{d}_1(S) + \widehat{d}_2(S)\right) + \widehat{i}_L(S)(1 - D_1 - D_2)$$
(4)

Equations (3) and (4) for dual input buck converter have two control inputs \hat{d}_1 , \hat{d}_2 and two disturbance inputs such as $\hat{V}_1 \& \hat{V}_2$ to regulate the output. In order to obtain a small signal model of MISO buck converter, the author replaced the terms in (3) & (4) with voltage sources, current sources, current and voltage dependent sources as shown in Figure 2. The primary input side of the small signal model obtained by perturbing the steady state currents across the switches S_1 and S_2 . The steady state average currents equations for two switches are as follows

$$I_{S_1} = D_1 I_L \tag{5}$$

$$I_{S_2} = D_2 I_L \tag{6}$$

The perturbed and Laplace form of equation (5), (6) is given as follows

$$I_{s_1}(S) = D_1 \hat{i}_L(S) + d_1(S) i_L$$

$$\widehat{I_{s_2}}(S) = D_2 \hat{i}_L(S) + \hat{d}_2(S) i_L$$

By considering all of these equations, the author successfully obtained the small signal model for MISO buck converter depicted in Fig. 2.



Fig. 2. Small signal model of DISO buck converter

From figure (2), it can be perceived the small signal model has two control inputs $\hat{d}_1 \otimes \hat{d}_2$ and two disturbance inputs $\hat{V}_1 \otimes \hat{V}_2$. The control inputs \hat{d}_1 and \hat{d}_2 can be controlled depending upon the user requirement. They can be controlled either to share optimal power between the inputs or for the purpose of maximum power point tracking (MPPT). The small signal model in figure 2 can help to obtain the transfer function required to achieve the selected control objective. In order to study the effects of perturbations in the duty cycle " D_1 " of switch S_1 , the transfer function GV_0 will be obtained. The transfer function GV_0 is the transfer function of control input \hat{d}_1 to the output voltage V_0 of the converter. GV_0 can be easily find out with the help of equations (3) & (4) by putting disturbance inputs $\hat{V}_1 \otimes \hat{V}_3 = 0$

$$GV_0 = \frac{V_0(s)}{\hat{d}_1(s)} \Big[\hat{d}_2 = \hat{V}_1 = \hat{V}_2 = 0 = \frac{V_p(1 - D_1 - D_2) - LSi_L}{S^2 LC + S \frac{L}{R} + (1 - D_1 - D_2)} \Big]$$

3. Proposed PWM Control Strategy & Controller Design

In this section, the author of this paper discuss a new but simple control strategy for the multi-input buck converter. This section also includes a flow diagram demonstrating the procedure from modelling to controller design as shown in figure 3. Depending upon the stability analysis, the author run the AC analysis of the multi-input converter in Ltspice. This step provided the information about the phase and gain of the converter. As the gain was more than 90 degrees but less than 180 degrees, therefore author selected the type-III amplifier. Using the steps introduced in K factor method [20], the author find out all the supporting values for the resistors and capacitors required for type-III compensator. In this control strategy, the output of the system given to a negative feedback controller, which has a reference voltage on its positive terminal. The output of the type-III compensator then compared with a saw tooth waveform for the pulse width modulation. At this stage, in order to control the two switches by changing the duty cycle a new factor called M introduced in the circuit. This value M will control the duty cycle of the second switch. With the help of M, the author can change the duty cycle of the second switch depending upon the requirements of the circuit resulting in an unlimited control strategy for this type of converters. The value of M is adjustable depending on the duty cycle required for the switches. During the simulation, we change the output load in order to check the response of the converter control to see how well it is responding to output load changes depicted in Fig 9.

In order to find the exact value, the formula for finding out M on should use the same formula as the voltage divider. The value of M will determine the value of the resistor, which will divide the duty cycle from the compensator. After reducing the voltage level to a required level, the output is then compared with a same saw tooth waveform carrier. Finally, it will be fed into the EXOR gate and then to switch one in order to control the other switch. In case of increasing the inputs of this multi-input converter, this

value of M will also relate and fit with future work. This gives a base for a new transfer function as follows

$$\frac{V_{O}(s)}{\hat{d}_{1}(s)} \Big[\hat{d}_{2} = \hat{V}_{1} = \hat{V}_{2} = 0 = \frac{V_{p}(1 - D_{1} - D_{2}) - LSi_{L}}{S^{2}LC + S \frac{L}{R} + (1 - D_{1} - D_{2})}$$
$$M = D_{1}/D_{2} \qquad \& \qquad D_{2} = \frac{D_{1}}{M}$$

The final equation for the transfer function for the dual input buck converter given in equation 9.

$$GV_{0} = \frac{V_{0}(s)}{\hat{d}_{1}(s)} \Big[\hat{d}_{2} = \hat{V}_{1} = \hat{V}_{2} = 0 = \frac{V_{p} \Big(1 - D_{1} - \frac{D_{1}}{M} \Big) - LSi_{L}}{S^{2}LC + S\frac{L}{R} + (1 - D_{1} - D_{2})}$$
(9)

Depending upon the user's requirement, we can change the value of M in order to change the duty cycle of the first switch to get more control over the output of the system. The overall controller design is as follows (Fig.3)



Fig. 3. DISO buck converter with control strategy

4. Simulation Results & Discussion

This section includes all the simulation results of the un-compensated transfer function as well as the voltage after using type-III compensator. Before starting the controller design, the author used the stability analysis in order to know whether selected topology was feasible or not. In this step, the author simulate selected topology in order to know about the phase and gain of the circuit. The phase & gain of the circuit given in Fig 4. The phase of the circuit was -168.37 degrees while the amplifier gain was - 8.564 DB at the crossover frequency of 3 KHz. Using the steps presented in K factor Method [20], the required boost was calculated i.e. 113.37 degrees. Depending on the required boost, the author selected the type-III compensator, as it is the only feasible compensator type for this kind of topology.









Fig. 5. Un-compensated Transfer Function



Fig. 7. Bode Plot of MISO Transfer function

Fig. 6. Inductor Current

Before designing the controller, the un-compensated plot of the transfer function is also plotted in Matlab (Fig 5) in order to know about the exact characteristics of the circuit. The control objective of the circuit was to provide 24 volts at the output voltage. For this reason, the controller was designed and the circuit was contructed in Ltspice for simulation. While finalising the control strategy, it was found that the control circuit requires EXOR gate which will controll the switch S1 connected with the compensator while Switch S2 is directly controlled by the type 3 compenster. In Fig 10, the connection between the compensator and the PWM block, one should see the duty cycle of the switch S1 is limited in order to limit the duty cycle of switch 2.

This is done by introducing a new variable M whose value will determine the relationship of two duty cycles. In first case, we are taking the value of M = 0.5. The reference voltage V_{ref} was set to 3.3 volts with the help of two parallel resistors connected to the negative terminal of the amplifier. The output of the amplifier then compared with a saw-tooth waveform for performing pulse width modulation in order to control the switching of the circuit.



Fig. 8. 24 Volts output voltage with M=0.5

Fig. 9. 24 Volts output voltage with M=0.45

At this point, in order to vary the switching of first switch, the voltage divider rule is applied to change the duty cycle to half by placing same value resistors. In this way, there will be a switching cycle will change for the first switch as compared to switch 2. The negative feedback amplifier is controlling the duty cycles. In second case, we are taking the value of M=0.45 and repeat all the above procedure again. The effect of new value can be seen in Fig 11. The inductor current is in continous conduction mode as it was the main objective of the simulation shown in Fig 7. The output voltage and the effect of new variable M on the waveforms can be seen in Fig 9 & Fig 10. The Fig 9 satisfies the original control objective, as the main challenge was to control the voltage to 24 volts

5. Conclusion

This paper has introduced voltage mode control for multi-input dc-dc buck converter with a new control strategy including a new variable M. A small signal model & closed loop control to output transfer function required to control the multi-input converter in continuous conduction mode is presented. During control design, the type-III controller has been selected as it meets all the expectations and challenges of the closed loop control system. The closed loop control to output transfer function was simulated into computer simulations using Ltspice. The layout of the software prototype indicates that this topology is much more stable than other converter topologies in case of multi-input converters. The simulation of developed DISO transfer function is in MATLAB.



Fig. 10. Effect of new Variable M=0.5



Fig. 11. Effect of new Variable M=0.45

Conflict of Interest

The authors declare no conflict of Interest.

Author Contributions

This paper is contributed by three different authors. M M Alam (Author A) has prepared the small signal modelling of the DISO Buck converter. The author A went through a number of steps while completing this stage. It includes gathering all knowledge about mathematical steps involved in the calculations, cross-checking the results with the theoretical knowledge. The author A also finished the simulation part of the work where he went through a number of problems. At the end, the author A wrote the paper. During this whole procedure, the D C Lu (Author B) and Y Siwakoti (Author C) were supervising Author A. Whenever any problem arises during calculations, writing or simulation, the Author B and C helped Author A. Finally, all the results were double checked by the Author B and C. Author B and C also done the cross check of paper writing and provided help in the simulation of the idea. At the end, all the authors had found the final draft complete and ready for publishing.

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