Analytical model of wave energy converter based on water mass gravity force under regular wave to generate electricity

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Abstract

To date there is an inadequate information about the analytical model of wave energy converter based on water mass gravity force to generate clean energy. This work proposed an analytical model of wave energy converter based on that previously studied using physical model that were carried out in laboratory experiment at Hasanuddin University Indonesia. This model was derived from analytical of regular wave and converter interactions. Moreover, the proposed analytical model was validated by comparing the simulation result with the previous physical model laboratory experimental result. Analytical model simulation and physical model experiment results, showed a similar trend under wave height and period variation respectively, despite the fact that there were slightly different Power Take Off value. Furthermore, this proposed analytical model will be very useful in predicting the magnitude of electricity that can be generated by means of wave energy converter based on water mass gravity force. The analytical model indicated that the converter’s Power Take Off do not depend entirely on the wave height and period. However, the amount of generated power needed can be achieved by varying the number and volume of gravity weights respectively.

Keywords: Analytical model, wave energy converter, gravity force

1. Introduction

The use of ocean wave as an energy source has long been known since 18th century in Southern California, but has not experienced much development due to several failures [1]. Later on, after the fuel oil crisis due to the embargo of OPEC member countries in 1973-1974, attention to harness the alternative energy sources began to receive attention pioneered by US president Richard Nixon, during the energy crisis. Then, attention to the development of alternative energy sources, as well as the use of ocean wave energy, began to attract great attention from researchers [2].

A number of studies to convert the ocean wave energy to generate electricity have been carried out in recent years. Hansen et al utilizes the waves as a source of renewable energy by harnessing the movement up and down the waves to drive a hydraulic pump which is connected to the turbine to rotate the electric generator [3]. Kelly, et al proposes a combination of a floating wave power stations with wind power plant to cope the variations of the electrical energy produced [4]. O’Cathalin Studied the interaction of ocean wave energy converter with nonlinearities of the ocean waves using buoys connected to each other [5]. The subsequent wave conversion technology is harnessing the potential energy of the buoy up and down movement to move a permanent magnet generator vertical translation [6], Pecher proposed floating wave energy converter that should be able to survive in extreme waves conditions [7]. Wacher at al, suggest a theoretical model to predict the converter’s Power Take Off (PTO) that can be applied for regular and irregular wave [8]. Hansen, offered hydrostatic transmission topology model using generic components to predict the PTO efficiency generated by the wave energy converter [9].

However, the findings of the previous studies focus on capturing the buoyancy force of the ocean.
waves. The converter’s PTO rely on the difference between buoyancy force and gravity force of the buoy which is acting as the converter’s and waves interface. Therefore, this research proposed different approach by harnessing the water gravity force instead of wave buoyancy force. In other words, the proposed converter focusses on Newton force rather than Archimedes force.

To the best of our knowledge, there is no information in the literature regarding the analytical model of wave energy converter that using water mass gravity force. Therefore, the aim of the present work is to construct a comprehensive model to predict the proposed converter’s PTO based on water mass gravity force.

The PTO’s of the model simulation result under wave height and period variations are in line with physical model experiment results. Hence, the suggested analytical model in this work could become a useful tool to predict the amount of generated power by wave energy converter based on water mass gravity force.

2. Methodology

2.1. Wave-Converter interactions model

The purpose wave energy converter is how to capture wave energy when it moves from the crest to the trough. To achieve this goal, a unidirectional or one-way gear is used connected with gravity weight container filled with water (Fig. 1). The up and down movement of the container that connected to one-way gear along with the ups and downs of the waves will rotate the converter shaft in a unidirectional rotation [9].

![Fig. 1. Basic concept of wave energy converter based on water mass gravity force](image)

The size of converter’s components that were used in experimental studies in the laboratory consist of gravity weight made of plastic container; Length 28.8 cm, Width 23 cm and height 10 cm respectively. Moreover, the gear diameter 72 cm and counter weight mass is 0.328 kg. The conversion of wave energy into electricity is carried out by harnessing the down movement of gravity weight that will rotate the one-way gear. Furthermore, the one-way gear will rotate the converter’s shaft, fly wheel and finally drive the electric generator as illustrated in Fig. 2. In this experiment there were four gear-gravity weight pairs were employed.
The development of the analytical model of wave energy converters based on the gravitational force of water masses started by formulation of analytical interaction between wave and converter’s components. The first step is to determine the diameter and circumference of one-way gear used (Equation 2.1).

\[
G_c = 2\pi r
\]  

(2.1)

or

\[
G_c = \pi D
\]  

(2.2)

where;

- \( G_c \) = Gear circumference
- \( D \) = Gear diameter

Converter shaft rotation is determined by the circumference of the gears, wave height, number of gear-gravity weight pairs and gears spacing. Therefore, the minimum wave height required that enable the converter shaft to rotate \( 360^\circ \) is the product of gear circumference divided by the number of gear-gravity weight combinations.

\[
h_{\text{min}} \geq \frac{\pi D}{n_{gw}}
\]  

(2.3)
After the minimum wave height is determined, the next step is how to determine the maximum wave height that can be received by the proposed wave energy converter. Since, the converter is installed in a fixed construction, it is necessary to consider the high tide Still Water Level as a reference in determining the maximum wave height. Based on the illustration in Fig. 3, the maximum wave height that can be received by the converter is determined by the distance between the highest tide level and the converter shaft elevation.

Furthermore, the other variables that determine the performance of the proposed converter are wave wavelength ($\lambda$) and wave period ($T$). To ensure that the converter can always absorb energy in one wave cycle, the converter shaft length used must be arranged in such a way as to be longer or equal to the maximum wave length. Conversely, in setting the distance of gear spacing have to consider the minimum wave length. By setting the shaft dimensions and gear position this way, the converter shaft will continue to rate even though the wave period varies.

2.2. Converter’s shaft rotation speed modeling

The rotation speed of the converter shaft can be calculated based on the angular velocity equation shown in Equation (2.4)

$$\omega = \frac{2\pi}{t} \text{ (radial/second)}$$

(2.4)

When the gravity weight moves down following the wave movements as in the illustration in Fig. 1, the gear will cause point a to move to point b (Fig. 4) as far as $\alpha$ radians. Therefore, the wave height is equal to the distance travelled from point a to point b. Hence, the angle between point a and point b can be determined based on Equation 2.5 below.

$$\alpha = \frac{2h}{D} \text{ radian}$$

(2.5)

The angular velocity between point a and point b then is derived from the following Equation 2.6;

$$\omega = \frac{\alpha}{t} \text{ (radial/second)}$$

(2.6)

The time allocation for each gear is obtained from the ratio between the distance of the gear spacing ($G_s$) to the wave length multiplied by the wave period.

The gear spacing ($G_s$) is obtained by divide the wave length with the number of gear gravity weight pairs Fig. 5.

$$G_s = \frac{\lambda}{n_{gw}}$$

(2.7)

The time allocation in one wave cycle for each pair of gear gravity weight to rotate the converter shaft is the ratio of gear spacing with the wavelength multiplied by the wave period of Equation (2.8).

$$t = \frac{(G_s T)}{\lambda}$$

(2.8)
Therefore, the linear speed from point a to point b Fig. 4;

\[ V = \frac{H\lambda}{(G_s T)} \text{ m/sec} \]  \hspace{1cm} (2.9)

Fig. 5. Gear spacing of one-way gear and gravity weight pairs

The relationship between linear velocity and angular velocity is

\[ \omega = \frac{H\lambda}{(G_s Tr)} \text{ (revolution/second)} \] \hspace{1cm} (2.10)
\[ \omega = \frac{60 \lambda}{(G_s Tr)} \text{ (revolution/minute)} \] \hspace{1cm} (2.11)

2.3. Analytical model of converter’s shaft torque

Torque acting on converter’s shaft is the product of multiplication between the forces (F) multiplied by the length of the gear radius (r). Therefore, the force acting on the converter shaft is affected by the mass of gravity weight (M_{gw}), counter weight (m_c) and gear radius (r).

\[ \text{Torque} = r (M_{gw} - m_c)g \text{ (Newton meter)} \] \hspace{1cm} (2.12)

Equation 2.12 applies to a pair of gear-gravity weight. If a number of gear-gravity weights pairs are employed, then, the amount of torque acting along the converter shaft is the product of multiplication of the active gravity weight with Equation 2.12. Refers to Fig. 4, it shows that in one wave cycle only half of the gravity weight works. Therefore, the acting torque on converter’s shaft is;

\[ \text{Torque (}\tau\text{)} = 0.5n_{gw}r(M_{gw} - m_c)g \text{ (Newton meter)} \] \hspace{1cm} (2.13)

2.4. Analytical Model of Converter’s Power Take Off (PTO)

The power takes off is calculated based on the relationship between Torque (N.m), Power (Watt) and angular velocity (RPM) of the converter flywheel’s shaft.

The Power Take Off (kWatt) is yielded by multiplying torque (RPM) Equation 2.13 with angular velocity (RPM) Equation 2.11 divided by 9548.8 [11].

\[ \text{Power (kW)} = \frac{\text{Torque (N.m)} \times \text{Speed (RPM)}}{9548.8} \] \hspace{1cm} (2.14)

Or

\[ \text{Power (Watt)} = \frac{\text{Torque (N.m)} \times \text{Speed (RPM)}}{9.5488} \] \hspace{1cm} (2.14)
Therefore, the power takes off formula for wave energy converter based on water mass gravity force is expressed in equation 2.15 as follows;

$$\text{Power (W)} = \frac{\eta_c \eta_g n_{gw} \pi g H \lambda}{C_g T} (M_g - m_c) \text{(Watt)} \tag{2.15}$$

where;
- $M_g$ = Mass of gravity-weight (kg)
- $m_c$ = Mass of counter-weight (kg)
- $g$ = Gravity acceleration (m/s$^2$)
- $H$ = Wave height (m)
- $\lambda$ = Wave length (m)
- $C_g$ = Gear Spacing (m)
- $T$ = Wave period (sec)
- $n_{gw}$ = Number gear-gravity weight pairs
- $\eta_c$ = Converter efficiency
- $\eta_g$ = Electric generator efficiency

2.5. Model validation

Model validation was carried out by comparing the proposed analytical model (Equation 2.15) and actual results of physical model laboratory tests at wave flume Faculty of Engineering, University of Hasanuddin Indonesia. The dimension of physical model Length 435 cm, width 26 cm and height 56 cm. The power take off harvested in the physical model experiment is the product of torque (measured using newton meter scale) multiply by flywheel rotation speed (RPM) measured by means of digital tachometer.

![Physical model experiment set-up](image)

3. Result and Discussion

3.1. Wave height variation

The simulation using theoretical models under wave height variation experiments are much greater than the measurements results obtained from physical models experiments in the laboratory. However, by changing the value of $\eta_c = 0.008502$ in equation 2.15, the simulation and experimental measurement showed similar results as presented in Table 1. Initially, the analytical model results are slightly higher then experimental result. However, after the wave height reaches 6 cm, the theoretical model result is very close to experiment result. At the wave height of 8.5 cm there was a surge in the measurement...
results. The surge is probably caused by the accuracy of measuring instruments, especially Newton meters, that was still using analog systems.

Table 1. Power take off base on experimental and analytical model simulation under wave height variations

<table>
<thead>
<tr>
<th>Wave Height (cm)</th>
<th>Wave length (cm)</th>
<th>Flywheel radial speed (RPM)</th>
<th>Power Take Off (Watt)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Physical Model</td>
</tr>
<tr>
<td>2</td>
<td>160</td>
<td>12.4</td>
<td>0.01</td>
</tr>
<tr>
<td>2.5</td>
<td>160</td>
<td>33.2</td>
<td>0.04</td>
</tr>
<tr>
<td>3</td>
<td>160</td>
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<td>0.14</td>
</tr>
<tr>
<td>4</td>
<td>160</td>
<td>81.8</td>
<td>0.26</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
<td>107.3</td>
<td>0.42</td>
</tr>
<tr>
<td>6</td>
<td>160</td>
<td>143</td>
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</tr>
<tr>
<td>7</td>
<td>160</td>
<td>190.4</td>
<td>1.20</td>
</tr>
<tr>
<td>8</td>
<td>160</td>
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<tr>
<td>8.5</td>
<td>160</td>
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<td>1.88</td>
</tr>
<tr>
<td>9</td>
<td>160</td>
<td>214.7</td>
<td>1.52</td>
</tr>
</tbody>
</table>

3.2. Wave period variations

Table 2. Analytical model simulation and experiment result under wave period variations

<table>
<thead>
<tr>
<th>Wave height (cm)</th>
<th>Wave period (second)</th>
<th>Wave length (cm)</th>
<th>Flywheel radial speed (RPM)</th>
<th>Power Take Off (Watt)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Physical Model</td>
</tr>
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<td>5</td>
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<td>93.4</td>
<td>0.54</td>
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<tr>
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<td>0.78</td>
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<tr>
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<tr>
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<td>1.2</td>
<td>160</td>
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<td>1.75</td>
</tr>
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</table>

Similarly, with wave height variation experiment, the analytical model proposed in this paper can be used to predict the wave energy converter power take off under wave period variation. However, Equation 2.15 need to be corrected to suit the actual experiment result. Comparison between theoretical model and the physical model measurement results shows a similar pattern, where the wave period is inversely proportional to the power produced (Table 2).

The proposed analytical model presented in this paper is able to predict the power take off (PTO) of wave energy converter based on water mass gravity force. The novelty of this analytical model is lay on its capability to calculate electric power that can be generated based on wave height, wave period, number of gravity weight and gravity weight mass. The power produced is not depend solely on significant wave height and period but it depends on the number and volume of gravity weight used.

4. Conclusion

The results of calculations using the analytical model of wave energy converter based on the water mass gravity force shows similar pattern with the physical model experiments that carried out in the
laboratory. Therefore, the analytical model presented in this paper can be used as a tool to predict the electric energy that can be generated by means of the proposed wave energy converter model, based on a given significant wave height and period in certain area. Moreover, the analytical model presented in this paper was validated under laboratory generated regular wave. Therefore, further studies are required to construct medium scale or full-scale prototype in the real sea environment, variation of converter components and development of custom design low rpm generator.

5. Acknowledgments

This research was supported by Pusdiklat Industri Ministry of Industry Republic of Indonesia and Politeknik ATI Makassar at Makassar City, South Sulawesi Indonesia. The authors would like to thank the research members for their kind assistance.

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