A Novel mathematical model to control power of photovoltaic system

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Abstract

This paper presents a novel approach to complete the mathematical model for photovoltaic system. Corresponding to each type of panel made from semiconductors, the relation between the diode factor and temperature at p-n junction is added into the mathematical model. Using calculation results from the iterative and bisectional technique to determine maximum power point, the system of equations converting some parameters from standard test condition to any working condition, datasheet published by manufacturers, the mathematical model is tested to ensure that the diode factor only depends on temperature at p-n junction. Moreover, the least square method is used to create the function representing this relation from only some pair-values. An experimental control system is designed by combining the proposed mathematical model and a voltage controller in this paper to drive value of voltage at input terminals of a DC/DC buck converter to value of voltage at maximum power point. Experimental results showed the correctness between calculation and datasheet, high accuracy of the proposed mathematical model for any structure of photovoltaic system that helps to simulate the reaction before having actual implementation and harness maximum power from photovoltaic systems.

Keywords: Diode factor, least square method, maximum power point, mathematical model, photovoltaic system, p-n junction, voltage control.

1. Introduction

Photovoltaic system (PVs) made from semiconductors has become the most popular generation in renewable sources. Almost cells of the PVs are fabricated by a p-n junction in a thin wafer of semiconductor. Almost researches in control field are interested in its mathematical model because it affects to the process of simulating and experimenting before having actual implementation or choosing the best method to harness all energy from this generation. To represent the nonlinear behavior, single-diode and multi-diode exponential models often used as equivalent circuits of the PVs [1], [2]. Main elements in these models are: a current source exhibiting the cell photo-generated current when having incident light, series and parallel resistors representing power losses at contact layers or the resistance of the material, diodes having reversed polarity to characterize the current flowing through the poor region between the p-n junction. Although the multi-diode models can help to enhance the accuracy, they make difficulties for modeling because of using too many parameters in a mathematical model [1]. So, the single-diode model, used in this research, is the most popular model to describe the reaction of the PVs at any working condition, where each working condition has been specified by values of power of electromagnetic radiation (G) and temperature at p-n junction (T).

In single-diode model, there are some specified parameters describing the v-i characteristic such as open-circuit voltage V_{OC}, short-circuit current I_{SC}, photo-generated current I_{ph}, series resistance R_S, parallel resistance R_p, reversed saturation current I_0, thermal voltage V_t, diode factor n. Value of I_{ph}, R_S, R_p, I_0, V_t at standard test condition (STC), that has $G = 1000$ W/m^2 and $T = 25^0$C, was showed the method

to determine by solving system of nonlinear equations and using Newton-Raphson or Gauss-Seidel method [3]-[5]. Moreover, the variable law for these parameters corresponding to value of G and T at any working condition was determined and published in many previous studies by laboratorial experiments. Specially, diode factor n (or diode ideality factor as called in some researches) is often assigned a constant value that is 1 or 2. Above analysis shows that almost specified parameters in the single-diode model except n depend on the pair-values of (G, T). It can make less accurate to build v-i and v-p curves, evaluate the position of working points on these curves, simulate and test the performance of the PVs before bringing it out into real applications. Furthermore, almost researches studying PVs in the power system have been interested in its maximum power point (MPP) at any working condition. So, it needs to completely have evaluations about the effect of n to value of power at MPP. It means that the correction of n must be worked to coincide the calculated value at MPPs in the mathematical model and their value in datasheets provided by manufacturers.

To evaluate the relation of n and (G, T), it is necessary to combine the system of equations that converts value of almost parameters from STC to any working condition and mathematical tools to determine accurate value of parameters at MPP using the mathematical model. The iterative and bisectional technique (called IB technique) proposed recently can help to exactly calculate parameters at MPP by only using mathematical tools [5]. This technique overcame the difficult problem to find the extreme point when solving $dp_{pv}/dv_{pv} = 0$ because i_{pv} is a nonlinear function of both i_{pv} and v_{pv} variables.

Due to the impact of both G and T variables to v-i and v-p characteristics, it must be evaluated the individual effect of each variable to the accuracy of currently mathematical model to power at MPPs. Moreover, the variation of these variables is in a wide range and the characteristics of each type of the PVs are very different for each manufacturer. So, it is necessary to build a clear method to determine the function representing the relation between n and (G, T). This work can help to accurately model, simulate or calculate parameters at MPP.

Basing on single-diode model, parameters at MPP determined by using the IB technique and system of equations converting value of specified parameters from STC to any working condition as represented in section II, a proposed algorithm and an example will be represented in section III to realize the true relation of n and each variable G or T. Furthermore, the least square method will be also introduced and applied in this section to create the function that represents above relation only using some pair-values. In section IV, an experimental system will be introduced to verify the accuracy of the proposed model of the PVs. The final section represents some achievements and contributions in this paper.

2. Mathematical Description of the PVs and the IB Technique to Determine Parameters at MPP

2.1. Describe the mathematical model

The equivalent circuit of the PVs is represented by a single-diode exponential model as shown in Fig. 1. Representing equation (1) and (2), we have v_{pv}-i_{pv}, v_{pv}-p_{pv} curves corresponding with each pair-value (G, T). Each curve always exists a peak point called MPP and it divides two curves into two sides as shown in Fig. 2 [1]-[4].
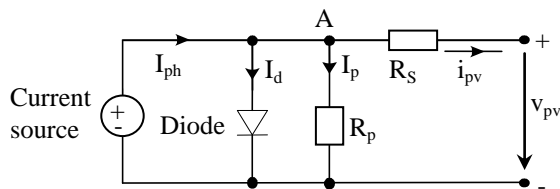

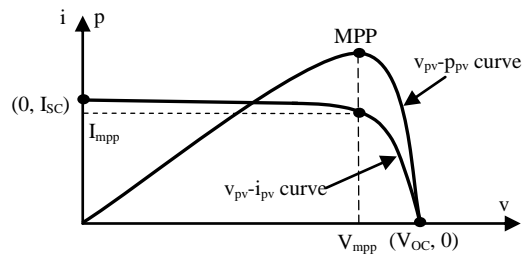Fig. 1. Equivalent circuit of the PVs

Fig. 2. v_{pv}-i_{pv} and v_{pv}-p_{pv} curves of the PVs

Apply Kirchhoff law equation at node A:

$$i_{pv} = I_{ph} - I_0 \left\{ \exp\left(\frac{v_{pv} + i_{pv}R_s}{nV_t} \right) - 1 \right\} - \frac{v_{pv} + i_{pv}R_s}{R_p} \tag{1}$$

Instantaneous power generating from PVs:

$$p_{pv} = v_{pv}i_{pv} \tag{2}$$

The system of equations converting value of I_{ph}, I_{SC}, V_t, R_p, R_S from STC to any working condition is represented by (3) [5].

$$\begin{cases} I_{ph}\big|_{G,T} = \dfrac{G}{G_{stc}} \{ I_{phstc}[1 + C_{TI}(T - T_{stc})] \} \\[2mm] I_{SC}\big|_{G,T} = I_{SCstc} \left[\dfrac{G}{G_{stc}} + C_{TI}(T - T_{stc}) \right] \\[2mm] V_t\big|_{G,T} = V_{tstc} \dfrac{T}{T_{stc}} \\[2mm] V_{OC}\big|_{G,T} = V_{OCstc}[1 + C_{TV}(T - T_{stc})] + V_t \ln \dfrac{G}{G_{stc}} \\[2mm] R_p\big|_{G,T} = R_{pstc} \dfrac{G_{stc}}{G} \\[2mm] R_S\big|_{G,T} = R_{Sstc} \end{cases} \tag{3}$$

where, values of symbols having "stc" are defined in STC,

2.2. IB technique to determine MPP at STC

Because of the complexity of equation (1), V_{mpp} and I_{mpp} can not be identified by solving equation $dp_{pv}/dv_{pv}=0$. Using detective technique for identifying pair-value $(v_{pv}(i), i_{pv}(i))$ at the ith step and bisectional technique by observing the movement of working points in a v_{pv}-p_{pv} curve as presented in Fig. 3 (continuous arrow for present direction, dash arrow for next direction), IB technique was proposed to identify MPP. It has two stages: the first one is that moves forward normally and the second one is that bisects as represented in Fig. 4 [5]-[7]. To ensure the convergence for this algorithm, ΔV should be chosen smaller than $(V_{OC} - V_{mpp})$.

a. Case 1 $(p_{pv}^{(i)} < p_{pv}^{(i+1)} < p_{pv}^{(i+2)})$

b. Case 2 $(p_{pv}^{(i)} < p_{pv}^{(i+1)}, p_{pv}^{(i+1)} = p_{pv}^{(i+2)})$

c. Case 3 $(p_{pv}^{(i)} < p_{pv}^{(i+1)}, p_{pv}^{(i+1)} > p_{pv}^{(i+2)})$
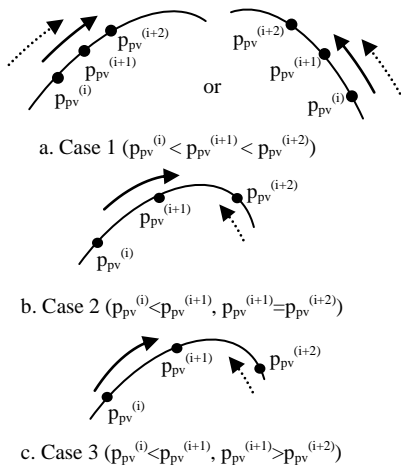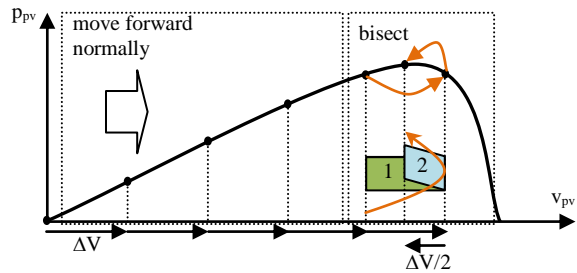
Fig. 3. Moving statement of working points

Fig. 4. Two stages of the IB technique

where: ΔV is value of voltage step.

The algorithm to determine pair-value $(v_{pv}^{(i)}, i_{pv}^{(i)})$ on each v-i curve corresponding to each pair-value (G, T) is represent in Fig. 5. The algorithm using the IB technique to identify MPP for PVs at any working condition is presented in Fig. 6 [5].
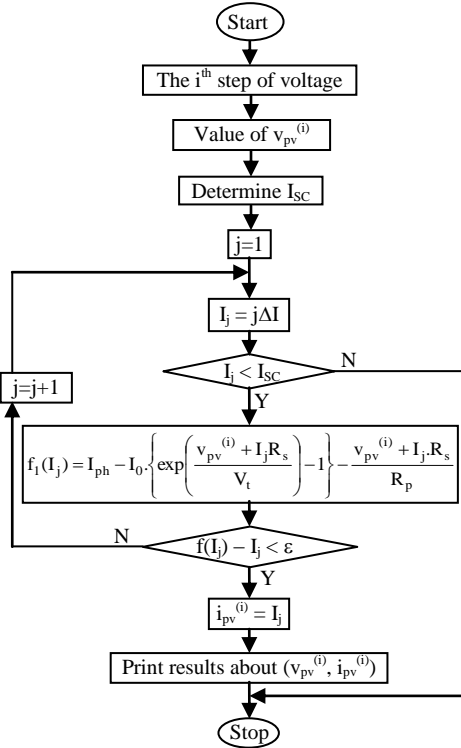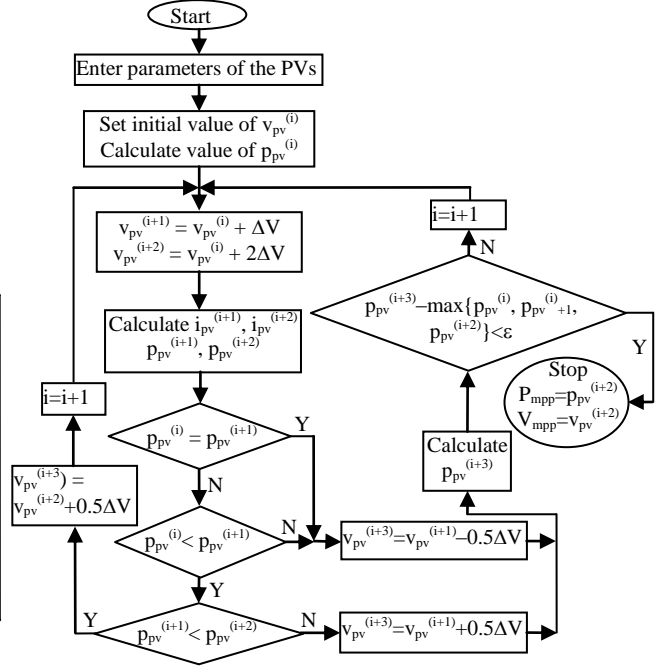
Fig. 5. Algorithm to determine pair-value $(v_{pv}(i), i_{pv}(i))$　　　Fig. 6. Algorithm using the IB technique to determine MPP

3. Completely Mathematical Model of the PVs

3.1. Propose an algorithm to test the effect of n to currently mathematical model

On the currently mathematical model, we can see that values of almost specified parameters vary in real working condition corresponding to the variation of pair-value (G, T) and can be defined through laboratorial experiments whereas n has been assigned a constant value from 1 to 2. Thus, n is given a value of 1 in this research for the 1st case (G varies, $T = T_{stc} = 25^0C$) and the 2nd case (T varies, $G = G_{stc} = 1000$ W/m^2) as represented in Fig. 7.

Applying above algorithm to test a SV-55 panel (a production of Schott - Germany), the value of parameters published by the manufacturer and results of Newton - Raphson method is represented in Table 1.

Table 1. Parameters of a SV-55 panel

| Type of parameters	Symbol	Value
Short-circuit current	I_{SC} (A)	3.25
Open-circuit voltage	V_{OC} (V)	22.14
Voltage at MPP	V_{mpp} (V)	18.4
Current at MPP	I_{mpp} (A)	3.06
Temperature coefficient of I_{SC}	C_{TI} (mA/^{0}C)	4.7
Temperature coefficient of V_{OC}	C_{TV} (mV/^{0}C)	-0.743
Temperature coefficient of power	C_{TP} (%/^{0}C)	-0.451
Photo-generated current	I_{ph} (A)	3.2502
Reversed saturation current	I_0 (A)	1.623×10^{-8}
Thermal voltage at p-n junction	V_t (V)	1.141
Series resistor	R_S (Ω)	0.151
Parallel resistor	R_p (Ω)	1675.9
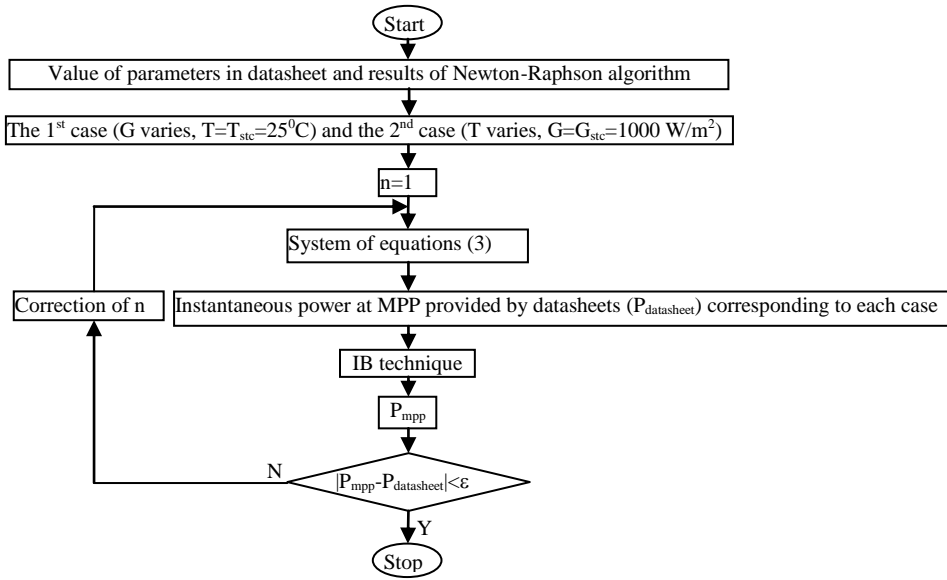
Fig. 7. Algorithm to test the effect of n to parameters at MPP

Received results corresponding to the 1st and 2nd cases are represented in Table 2.

Table 2. Received results corresponding to the 1st and 2nd cases

		The 1st case (G varies, T=25^{0}C)					The 2nd case (T varies, G=1000 W/m^2)				
		1000	800	600	400	200	25	35	45	55	65
P$_{mpp}$(W)	Calculated	55	43.7	32.5	21.4	10.1	55	57.4	59.5	61.55	63.74
	Datasheet	55	43.8	32.6	21.4	10.2	55	52.4	50	47.6	45.1

The calculated and published values are only the same values in the 1st case. In the 2nd case, calculated results increase while published values decrease. It means that value of n is necessary to be adjusted, where it only depends on value of the T.

3.2. Propose a new method to build the n(T) function

The purpose of proposing a new method is to provide an exact value of n corresponding to the value of T. In many mathematical tools, the least square method is a suitable method because it only uses some pair-values (n, T) to evaluate errors between calculated values and published values. The estimative process shows that the relation between n and T can be represented by the form of the second order function as (4):

$$n = a + bx + cx^2 \tag{4}$$

where: x= T - T$_{stc}$

Each error value at the ith sample can be determined by (5):

$$\varepsilon_i = a + bx_i + cx_i^2 - n_i \tag{5}$$

where, i=(1÷k) and k is the number of sample.

The objective function derived from the least square method by (6):

$$f(a,b,c) = \sum_{i=1}^{k} \varepsilon_i^2 = \varepsilon_1^2 + \varepsilon_2^2 + ... + \varepsilon_k^2 \rightarrow \min \tag{6}$$

To achieve the objective function, we have (7):

$$\frac{\partial f}{\partial a} = 0, \quad \frac{\partial f}{\partial b} = 0, \quad \frac{\partial f}{\partial c} = 0 \tag{7}$$

Value of a, b and c coefficients is determined by (8) [8]-[10]:

$$\begin{cases} ka + b\sum_{i=1}^{k} x_i + c\sum_{i=1}^{k} x_i^2 = \sum_{i=1}^{k} n_i \\ a\sum_{i=1}^{k} x_i + b\sum_{i=1}^{k} x_i^2 + c\sum_{i=1}^{k} x_i^3 = \sum_{i=1}^{k} n_i x_i \\ a\sum_{i=1}^{k} x_i^2 + b\sum_{i=1}^{k} x_i^3 + c\sum_{i=1}^{k} x_i^4 = \sum_{i=1}^{k} n_i x_i^2 \end{cases} \tag{8}$$

This method can help to calculate value of these coefficients for any panel or any type of the PVs. Applying to calculate the function for SV-55 panel, we have (9):

$$n(T)_{SV-55} = 1 - \frac{91}{10000}(T - T_{stc}) + \frac{1}{20000}(T - T_{stc})^2 \tag{9}$$

Retest the accuracy of power at MPP after using (8) to complete the mathematical model in the 2nd case, received results are represented in Table 3.

Table 3. Received results after using n(T) function

T		25	35	45	55	65
P_{mpp} (W)	Calculate	55	52.3	50	47.6	45
	Datasheet	55	52.4	50	47.6	45

After having the correctness of mathematical model for PVs, we can see that the calculated and published values are the same at any working condition. It expresses the role of n(T) function to make higher accuracy for the mathematical model and the parameters at MPP.

4. Verification on an Experimental System

4.1. Structure and devices on the experimental system

The structure of the experimental model is shown in Fig. 8. Devices in the experimental system are represented in Fig. 9.

This system has four main blocks: SV-55 panel for the generation block, DC/DC buck converter playing an adjustable load for the first block by changing control signal CS_1, a dynamic load to hold the voltage at output terminals of the converter at a constant value by changing control signal CS_2, measuring center to collect all measured parameters such as G, T, output terminals of the PVs (i_{pv}, v_{pv}), voltage v_{dc} at output terminal of the DC/DC converter, current i_{load} through the dynamic load.

SV-55 panel is also used in this system. PYR-BTA, a pyranometer of Vernier, is used to have information about G. LM35, a temperature sensor of National Semiconductor Corporation placed at the back of the SV-55 panel, is used to have information about T.

This system is designed and operated in Viet Nam to verify the accuracy of the proposed mathematical model for PVs. It means that instantaneous voltage (v_{pv}) at output terminals of the PVs is controlled to hold the expected value (V_{mpp}) that is determined by the IB technique and proposed mathematical model. If instantaneous power generating from PVs and the value of power (P_{mpp}) calculated by the IB technique is coincident, the proposed mathematical will be exact.
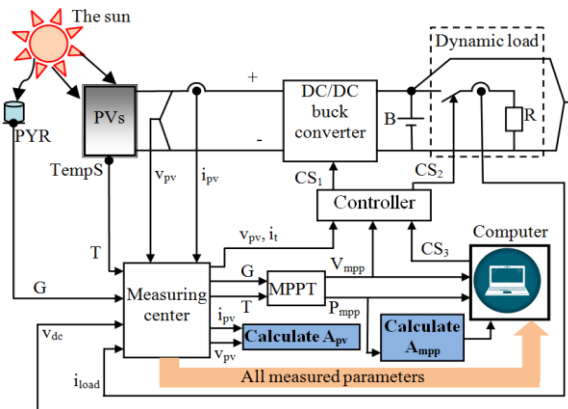

Fig. 8. Structure of experimental model

Fig. 9. Devices in the experimental system

4.2. Experimental results

Two samples were done by using the experimental model to verify the proposed model. The first sample was done from 8h45'58" to 8h49'18 and the second sample was done from 10h09'28" to 10h12'48 on June 20, 2018. In these samples, the variation of G and T is represented in Fig. 10 and Fig. 11.
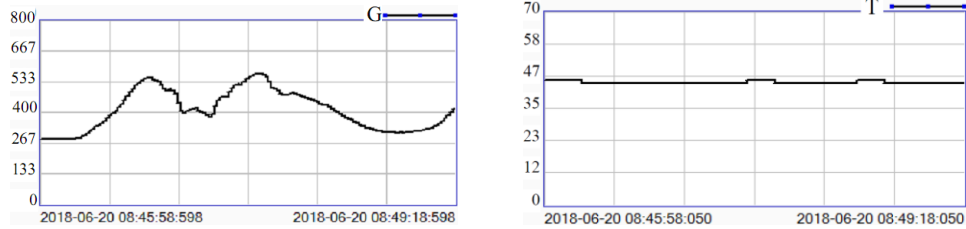
a. G diagram   b. T diagram

Fig. 10. Variation of G and T in the first sample
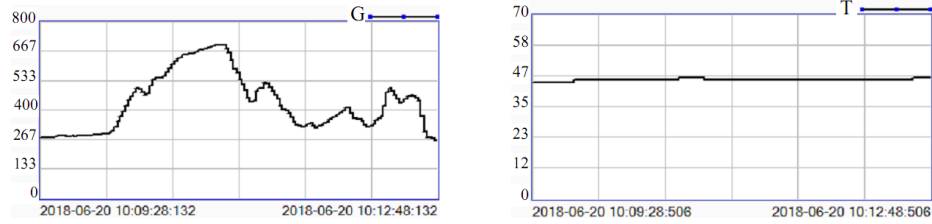
a. G diagram   b. T diagram

Fig. 11. Variation of G and T in the second sample

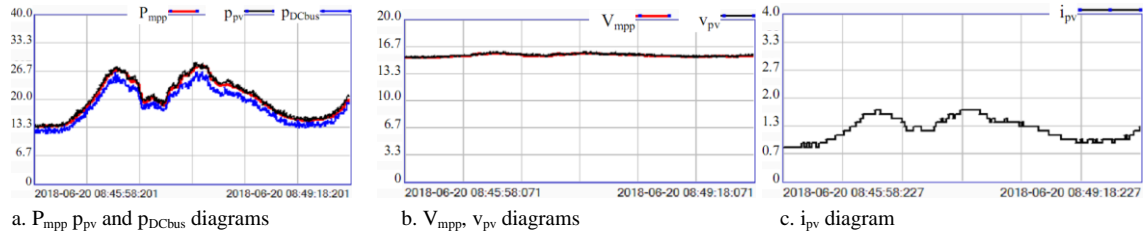The experimental results in two samples are represented in Fig. 12 and Fig. 13.

a. P_{mpp} p_{pv} and p_{DCbus} diagrams   b. V_{mpp}, v_{pv} diagrams   c. i_{pv} diagram

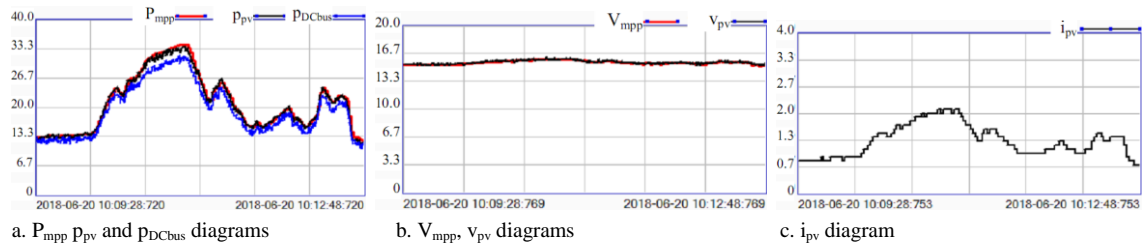Fig. 12. Experimental results in the first sample

a. P_{mpp} p_{pv} and p_{DCbus} diagrams   b. V_{mpp}, v_{pv} diagrams   c. i_{pv} diagram

Fig. 13. Experimental results in the second sample

where:

In Fig. 12a and Fig. 13a, the red diagrams represent value of power at MPP calculated by the IB technique and the black diagrams represent value of instantaneous voltage at the output terminals of the PVs. Blue diagrams depict the power going out the DC/DC buck converter that battery receives (p_{DCbus}).

In Fig. 12b and Fig.13b, the red diagrams represent value of voltage at MPP calculated by the IB technique and the black diagrams represents value of instantaneous power generating from PVs.

In both samples, G varied vigorously and T had a constant value. Received results show that when v_{pv} was controlled to track V_{mpp} at any time (the red curves and the black curves in Fig. 12b and Fig. 13b were

coincident), current generating from the SV-55 panel was also nearly coincident with the variation of G. Multiplying current and voltage, the instantaneous power diagrams generating from PVs tracked the P_{mpp} diagrams quite accurately although they were delayed that were caused by the controller and measured devices. Moreover, the p_{DCbus} diagrams also tracked simultaneously and were smaller than p_{pv} diagrams due to the power loss in the DC/DC buck converter. The coincidence between instantaneous power and calculated power in the experimental model proved the accurate correctness of mathematical model for PVs. It can help us affirm the effectiveness of using n(T) function to complete the mathematical model and harness MPP from PVs.

5. Conclusion

This paper proposes an algorithm to individually test the effect of G or T to the value of parameters at MPP at any working condition by combining the currently mathematical model of the PVs and the IB technique. It can be applied for any panel or any type of the PVs from any manufacturers. Received results show that n only depends on T.

This paper contributes a novel approach to complete the mathematical model for PVs. By applying the least square method, the n(T) function can be determined for any structure of the PVs from some specified pair-values (n, T). In theory, n(T) function helps parameters at MPP coincide with values published by manufacturers and enhance the accuracy of mathematical model for PVs. The propose of n(T) function is a good ideal for deep researches about PVs.

An experimental model was designed to verify the proposed mathematical model. In this model, instantaneous value of v_{pv} was controlled through the DC/DC converter to reach value of V_{mpp} at any time. Experimental results showed the coincidence of calculated and instantaneous values. Because of experimenting in real conditions in Thai Nguyen province, Viet Nam for a type of panel, the results proved the correct approach for any structure of the PVs when the mathematical model was fulfilled by adding the n(T) function. They also consolidated the reliability of theoretical contributions. With the successfulness of the proposed mathematical model, the reaction of the PVs can be evaluated more accurately through simulation process and the parameters at MPP can be precisely determined to provide to the controller to harness maximum energy from PVs at any real working condition. The success of this research brings out a new approach to model and control for any capacity of generation in smart grid.

Acknowledgements

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