Magnetic vector potential analysis for new design of transformer shape with V-connection in railway system

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Abstract

This paper presents a new design of transformer shape core with used in railway system via analyzed the magnetic vector potential in the form of partial differential equation. Finite element method is used to simulate three-dimensional system. This paper focuses on the magnetic vector potential behavior on core transformer with V-connection, while the transformer was supplied voltage from three phase system. The unbalance input voltage is divided to three case studies in this paper: C-A couple phase, A-B couple phase and B-C couple phase from three-phase system. In addition, a design of transformer shape core occurred on the corner that is the intersection between vertical core and horizontal core with 5mm radius curves. The simulation results compare magnetic vector potential between the original shape of transformer core and the new design of transformer core even though the supplied voltage are unbalanced. Nevertheless, the results shown that the new design of transformer core is more highly uniform a magnetic vector potential distribution in the transformer core and suitable for guideline for improving the shape of transformer core to reduce the core loss.

Keywords: Transformer design, V-connection transformer, magnetic vector potential, finite element method, railway system

1. Introduction

The transportation system is important connecting systems. There are many types such as the air freight, sea freight and rail transportations. The rail transportations are the most comfortable system in metropolitan society and a main public transportation system in many countries due to the great advantages related to deliver many people to anywhere and reduced the time for travelling. The electric power qualities to support the electric railway system must be high efficiency even the electrical equipment that is the part of the railway system such as transformer converter system and supplied sources etcetera. In case of using the rail transportations affecting the main electrical system, which unbalance system since working of the electric rail system was the single-phase load while the power supply was the three-phase system, which regulated the primary voltage through the primary side of autotransformer with V-connection. Therefore, this paper focused on the design of a shape transformer with V-connection for improving a performance of the transformer while used in the railway system with analyzed the magnetic vector potential. However, the result would be practical in term of improving the voltage of three phase main system will be balance.

2. Catenary Autotransformer

The electric energy of the rail system is supplied with high voltage to catenary feeder substations where the voltage is reduced to a suitable level and fed to the railway catenary conductors to be used by
locomotives and trains. The overhead catenary system is the supplied system of the railway system consisted of the bare wire conductors connected to the insulator which the current through the pantograph into the train’s propulsion system and became a closed loop system [1]. Electricity will flow through the rail or the fourth rail, which is grounded as follow the Fig. 1. The overhead power supply is usually connected to a high-voltage system to reduce the loss of power transmission over long distance [2-3].

For Thailand, electric railways using this system such as an Airport Rail Link (ARL) contains with substation sub power station electric multiple unit pantograph and catenary wire. Catenary autotransformer is the one of electrical equipment in the rail system, which is used in modern high power railway catenary systems fed with two phases with 180° phase shift with the midpoint connected between two phases and the secondary voltage winding between the catenary phase and earth return conductor [4]. The secondary voltage is the catenary voltage again the earth and the primary voltage is two times

![Fig. 1. Autotransformer for electric railway system.](image)

3. V-Connection Transformer

V-connection transformer, each of the two electrical substation transformers could be connected a different phase of the primary side. The completely supplied catenary is thus divided into sectors of lower length, which are separated from neural sections. Both of the electrical substations and at the halfway. The secondary voltages are out of phase with each other 2π/3 [5]. The equivalent supplied single-phase load corresponds approximately to the higher powers of a traction load. Its main drawback is that if a traction transformer fails, another one should cross phase supply while the three-phase supply will be interrupted. The diagram of a V-connection all three cases of supply from C-A, B-C and A-B couple phase of main system shown as the Fig. 2.
Fig. 2. V-connection transformer double phase supplied from main system (a) supply voltage from C-A (b) supply voltage from B-C (c) supply voltage from A-B.

This paper has considered in three schemes of the V-connection transformer is C-A, A-B and B-C which are coupled phase of main three-phase system supplied with primary side of autotransformer can be shown as the Fig. 3.

Due to this paper has considered the autotransformer with V-connection. Therefore, a secondary current can explain by (1) [6].

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
-a^2/Z_{eq} & 0 \\
0 & -a^2/Z_{eq}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
+ 
\begin{bmatrix}
aV_{AB} \\
aV_{BC}
\end{bmatrix}/Z_{eq}
\]

(1)

On the other hand, a primary current can explain depends on couple phase supplied condition where C-A supplied current, A-B supplied current and B-C supplied current can explain by (2) – (4), respectively [7-8].

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
-1 & -1 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

(2)

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
1 & 0 \\
-1 & -1
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

(3)

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} =
\begin{bmatrix}
-1 & -1 \\
0 & 1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

(4)
This paper considered the autotransformer with the rated voltage on secondary 25kV single-phase system and 115kV three-phase system supplied with primary side [9]. In addition, the equivalent impedance of the autotransformer from open circuit test and short circuit test is 0.3159+j11.1200 Ω and constant of all case studies.

4. Mathematical Modelling for Magnetic Vector Potential and Simulation Parameters

Finite element method is the most efficient numerical technique [10] for solving the partial differential equations (PDE) such as electromagnetic problem, temperature rise and heat transfer problem [11]. In terms of electromagnetic problems, mostly differential equation starting from magnetic vector potential form as follows (5) [12].

\[ \nabla \times \left( \frac{1}{\mu} \nabla \times A \right) + \sigma \frac{\partial A}{\partial t} = J_0 \]  

(5)

where, \( A \) is magnetic vector potential (S/m)
\( \sigma \) is electrical conductivity of material (Mho/m)
\( \mu \) is permeability of material, while permeability of free space is \( 4\pi \times 10^{-7} \text{ H/m} \)
\( J_0 \) is electrical current density (A/m\(^2\))

Vector identification properties as (6).

\[ \nabla \times (\nabla \times A) = \nabla (\nabla \cdot A) - \nabla^2 A \]  

(6)

Consequently, the (5) can be written to the magnetic vector potential equation while \( \nabla \cdot A \) is zero as the \( A \) properties. Therefore, the magnetic vector potential as follows (7).

\[ \nabla^2 A - \mu \sigma \frac{\partial A}{\partial t} = -\mu J_0 \]  

(7)

This paper has considered the problem in three dimensions therefore, the magnetic vector potential equation in \( x, y \) and \( z \) direction follows in (8).

\[ \frac{1}{\mu} \frac{\partial^2 A}{\partial x^2} + \frac{1}{\mu} \frac{\partial^2 A}{\partial y^2} + \frac{1}{\mu} \frac{\partial^2 A}{\partial z^2} - \sigma \left( \frac{\partial A}{\partial t} \right) + J_0 = 0 \]  

(8)

This paper has considered the system governing by using time harmonic mode and representing the magnetic vector potential in complex form therefore [13],

\[ \frac{\partial A}{\partial t} = j\omega A \]  

(9)

where, \( \omega \) is the angular frequency (rad/s).

From the (9), substituting the complex form to (8) can be converted to an alternative form as follows (10).

\[ \frac{1}{\mu} \frac{\partial^2 A}{\partial x^2} + \frac{1}{\mu} \frac{\partial^2 A}{\partial y^2} + \frac{1}{\mu} \frac{\partial^2 A}{\partial z^2} - j\sigma \omega A + J_0 = 0 \]  

(10)

The current density determines from the current quantities through the area of conductors can define by (11), where \( N \) is turn number of conductors (turn), \( I \) is current quantities (A) and \( A \) is area of conductors (m\(^2\)).

\[ J_0 = \frac{N \times I}{A} \]  

(11)
Fig. 4. Dimension of autotransformer (a) core of transformer (b) top view (c) right view.
Applying the finite element method for solving the PDE as (10) follow these steps.

First, determined both of nodes and elements of discretization the system. The general of element in three dimensions is tetrahedral which has number of nodes is 23792 and the number of elements is 136846 can be shown in Fig. (5).

Second, formulating the interpolation function of each element in three dimensions is derived from the Maxwell’s equations directly by using Galerkin methodology, which is the definite weight residual technique for which the weighting functions are similar as the shape functions. According to this method, the magnetic vector potential is expressed as follows (12) [14].

\[
A(x, y, z) = A_iN_i + A_jN_j + A_kN_k + A_lN_l
\]

(12)

where, \(N_i, N_j, N_k, N_l\) are the element shape functions of node \(i, j, k, l\) respectively, and \(A_i, A_j, A_k, A_l\) are the magnetic vector potential at node \(i, j, k, l\) respectively. The weighting functions that are similar as shape function can be written as (13).

\[
N_n = \frac{a_n + b_n x + c_n y + d_n z}{6V}
\]

(13)

where, \(n = i, j, k, l\) and \(V\) is the volume of each tetrahedral element, which defined as the (14).

\[
V = \frac{1}{6} \begin{vmatrix}
1 & x_i & y_i & z_i \\
1 & x_j & y_j & z_j \\
1 & x_k & y_k & z_k \\
1 & x_l & y_l & z_l
\end{vmatrix}
\]

(14)

And the positional coefficient defined by
\[ a_i = x_i \left( y_j z_k - y_k z_j \right) + x_k \left( y_i z_j - y_j z_i \right) + x_j \left( y_k z_i - y_i z_k \right) \]
\[ a_j = x_i \left( y_k z_i - y_i z_k \right) + x_k \left( y_j z_i - y_i z_j \right) + x_j \left( y_k z_i - y_i z_k \right) \]
\[ a_k = x_i \left( y_j z_i - y_i z_j \right) + x_j \left( y_k z_i - y_i z_k \right) + x_i \left( y_j z_i - y_i z_j \right) \]
\[ a_l = x_k \left( y_j z_i - y_i z_j \right) + x_i \left( y_k z_l - y_l z_k \right) + x_i \left( y_j z_l - y_l z_j \right) \]
\[ b_i = y_i \left( z_k - z_j \right) + y_k \left( z_j - z_i \right) + y_j \left( z_l - z_k \right) \]
\[ b_j = y_i \left( z_i - z_k \right) + y_i \left( z_k - z_j \right) + y_k \left( z_l - z_i \right) \]
\[ b_k = y_i \left( z_j - z_i \right) + y_j \left( z_i - z_j \right) + y_i \left( z_l - z_j \right) \]
\[ b_l = y_k \left( z_i - z_j \right) + y_l \left( z_j - z_l \right) + y_j \left( z_k - z_l \right) \]
\[ c_i = x_i \left( z_j - z_k \right) + x_j \left( z_k - z_i \right) + x_k \left( z_i - z_j \right) \]
\[ c_j = x_i \left( z_k - z_i \right) + x_k \left( z_i - z_k \right) + x_i \left( z_l - z_k \right) \]
\[ c_k = x_i \left( z_l - z_j \right) + x_j \left( z_j - z_l \right) + x_k \left( z_l - z_j \right) \]
\[ c_l = x_k \left( z_j - z_l \right) + x_j \left( z_l - z_j \right) + x_i \left( z_k - z_l \right) \]
\[ d_i = x_i \left( y_k - y_j \right) + x_k \left( y_j - y_i \right) + x_j \left( y_i - y_k \right) \]
\[ d_j = x_i \left( y_i - y_k \right) + x_i \left( y_k - y_i \right) + x_k \left( y_i - y_i \right) \]
\[ d_k = x_i \left( y_j - y_i \right) + x_j \left( y_i - y_j \right) + x_i \left( y_j - y_i \right) \]
\[ d_l = x_k \left( y_i - y_j \right) + x_i \left( y_j - y_k \right) + x_j \left( y_k - y_i \right) \]

Third, formulating the each element equations by integration by parts of the eq. (10) and substituting the approximate results in (10) which is equal residual function as follow (15).

\[
\frac{1}{\mu} \frac{\partial^2 A}{\partial x^2} + \frac{1}{\mu} \frac{\partial^2 A}{\partial y^2} + \frac{1}{\mu} \frac{\partial^2 A}{\partial z^2} - j\sigma \omega A + J_0 = R
\]  

(15)

where, \( R \) is the residual function and make an integration by parts using Gauss’s theory. Therefore, the residual function expresses as (16).

\[
\int \left( N_n \left[ \frac{1}{\mu} \left( \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} \right) - j\sigma \omega A + J_0 \right] \right) dV = 0
\]

(16)

From eq. (16) can be divided into three parts as follows (17) - (19).

\[
\int \left( N_n \left[ \frac{1}{\mu} \left( \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} \right) \right] \right) dV = 0
\]

(17)
\[
-\int_V (N_n \cdot [-j\omega A]) dV = 0
\]  
(18)

\[
\int_V (N_n [J_0]) dV = 0
\]  
(19)

Using the factorial formula with (17) – (19) and then each element equation can be written in term of matrix with 4x4 size depends on types of elements. Thus, from (17) became the permeability matrix depends on the coordination of the grid along with \( x, y \) and \( z \) direction, which showed in term of positional coefficient as follow (20), where \([K]\) is the permeability matrix of problem.

\[
[K]_{4x4} = \frac{1}{36\mu V} \begin{bmatrix}
 b_{11} + c_{11} + d_{11} & b_{12} + c_{12} + d_{12} & \cdots \\
 b_{21} + c_{21} + d_{21} & b_{22} + c_{22} + d_{22} & \cdots \\
 b_{31} + c_{31} + d_{31} & b_{32} + c_{32} + d_{32} & \cdots \\
 \vdots & \vdots & \ddots & \ddots \\
 b_{33} + c_{33} + d_{33} & b_{34} + c_{34} + d_{34} & \cdots \\
 \vdots & \vdots & \ddots & \ddots \\
 b_{41} + c_{41} + d_{41} & b_{42} + c_{42} + d_{42} & \cdots
\end{bmatrix}
\]  
(20)

For the (18) became the constant matrix depends on the constant of electrical conductivity and angular frequency as follows (21), where \([M]\) is the constant matrix of problem.

\[
[M]_{4x4} = \frac{j\omega \sigma V}{20} \begin{bmatrix}
 2 & 1 & 1 & 1 \\
 1 & 2 & 1 & 1 \\
 1 & 1 & 2 & 1 \\
 1 & 1 & 1 & 2
\end{bmatrix}
\]  
(21)

For the (19) became the load vector depends on the current density of transformer as follows (22), where \([F]\) is the load vector of problem.

\[
[F]_{4x1} = \frac{J_0 V}{4} \begin{bmatrix}
 1 \\
 1 \\
 1 \\
 1
\end{bmatrix}
\]  
(22)

Fourth, applying the boundary conditions in term of Neumann, both of the edges between oil and conductors and between oil and the frame of the transformer as Fig. 6.

Fig. 6. Define the boundary condition of the simulation system.
Fifth, solving the linear equation for calculation a result of magnetic vector potential. For the simulation parameters, the finite element method was used for solving the PDE in this paper. The parameters for simulation depends on the magnetic vector potential equation, which defined in (20)-(22). However, All of parameter simulation shown in the TABLE I. [15-16]

Table 1. Parameter of autotransformer simulation

<table>
<thead>
<tr>
<th>Materials constant</th>
<th>Material</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability</td>
<td>Steel</td>
<td>40,000</td>
</tr>
<tr>
<td></td>
<td>Copper</td>
<td>0.9999</td>
</tr>
<tr>
<td></td>
<td>Mineral oil</td>
<td>2.2000</td>
</tr>
<tr>
<td></td>
<td>Kraft paper</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>Free space</td>
<td>$4\pi \times 10^{-7}$ H/m</td>
</tr>
<tr>
<td>Electrical conductivity</td>
<td>Steel</td>
<td>$2.08 \times 10^6$ S/m</td>
</tr>
<tr>
<td></td>
<td>Copper</td>
<td>$5.80 \times 10^7$ S/m</td>
</tr>
<tr>
<td></td>
<td>Mineral oil</td>
<td>0.1080 S/m</td>
</tr>
<tr>
<td></td>
<td>Kraft paper</td>
<td>0.9999 S/m</td>
</tr>
</tbody>
</table>

5. New Design and Simulation Results

This paper has considered the magnetic vector potential in the core part of the V-connection of the autotransformer, which compare to the original core and new core design for analyze the core losses of autotransformer when operating in the railway system which supplied from unbalance three phase system. [17-19] Moreover, the new shape design of transformer core using the electromagnetic field behavior by making all of core corner curvature with 5 mm of the radius and the new design can be shown in Fig. 7.
For the simulation results, this paper has considered the magnetic vector potential of transformer core using finite element method and divided into three cases of supplying voltage from three-phase system, which unbalance couple phase: C-A couple phase, A-B couple phase and B-C couple phase. For the magnetic vector potential of transformer core which supplied from C-A, A-B and B-C couple phase of three-phase system shown as Fig. 8 – 10, respectively.
Fig. 8. A magnetic vector potential (Wb/m) for C-A couple phase (a) original core (b) new core design

Fig. 9. A magnetic vector potential (Wb/m) for A-B couple phase (a) original core (b) new core design
According to the Fig. 8 - 10, the magnetic vector potential distribution all over the transformer core can be described by the electromagnetic field theory that the magnetic field will have a decreased when is low changing rate of a magnetic vector potential that mean more highly uniform a magnetic vector potential distribution or have a low standard deviation. This simulation result can be helped to the guideline for designing the core of the transformer. For a standard deviation comparison of magnetic vector potential of transformer core can be shown in the Table II., which all couple phases of new transformer core have a standard deviation lower than original transformer core.

Table 2. A standard deviation of magnetic vector potential of transformer core

<table>
<thead>
<tr>
<th>Three-phase supplied to primary</th>
<th>Original transformer core</th>
<th>New transformer core</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>0.00267</td>
<td>0.00237</td>
</tr>
<tr>
<td>B-C</td>
<td>0.00609</td>
<td>0.00538</td>
</tr>
<tr>
<td>C-A</td>
<td>0.00507</td>
<td>0.00449</td>
</tr>
</tbody>
</table>

Fig. 10. A magnetic vector potential (Wb/m) for B-C couple phase (a) original core (b) new core design
6. Conclusion

This paper simulation via the finite element method for solving the partial differential equation of magnetic vector potential to design the core of autotransformer with V-connection while operating in the unbalance of three-phase system. In addition, the simulation presents a new design shape of transformer core for guideline to improve an efficient of transformer. The simulation results shown that the shape that has curvature of the corner of transformer core can more highly uniform a magnetic vector potential distribution. Therefore, designing technology of the core shape of the transformer is the alternating way for reducing the core loss of the transformer using in rail systems that leads to more energy saving in rail systems. The frequencies of materials of core transformer for core loss reduction will analyze as the future research. On the other hand, In terms of manufacturing there are many factors to determine.

Acknowledgment

This work was supported by School of Electrical Engineering, Institute of Engineering, Suranaree University of Technology.

References