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Time Domain simulation of PD Propagation in XLPE Cables Considering Frequency Dependent Parameters

Wei Zhang[a], Jian He[b], Lin Tian[b], Xuejun Lv[b], Hong-Jie Li[a]* and Chongxin Liu[a]

[a] *Xi'an Jiaotong University, No.28 Xianning West Road, Xi'an, 710049, China*
[b] *Linyi Power Supply Ltd., No.130 Jinque Hill Road, Linyi, 276000, China*

Abstract

Partial discharge (PD) detection and location is of great significance for the power cable insulation condition monitoring, where analysis of the PD propagation is needed. This paper presents the single-core model of cross-linked polyethylene (XLPE) cable considering the influence of skin effect and semi-conducting screens. To analysis the propagation of PD pulse in time domain, the model firstly is rational approximated using vector fitting method. Based on the fitting results, the π section lumped parameters model is then proposed. Examples on two cables with different permittivity approximating forms of semiconducting screens are presented.

Keywords: Semi-conducting screen, state-space model, vector fitting (VF), XLPE cable

1. Introduction

The semi-conducting screens can have significant impact on high frequency wave propagation in power cables [1], [2]. The high frequency cable model that can be used over 100MHz is firstly described in [1], and more accurate model for extruded cables is developed in [3]. A few works on the characterization of semi-conducting layers in cables are carried out, and the complex permittivity of the dispersive layers is usually represented by Cole-Cole and Debye models [4], [5]. Although there is good performance between the high frequency cable model and the actual measurements, it is still a tough problem to solve these models in time domain.

Vector fitting (VF) is a robust numerical method which can be applied to fitting of measured or calculated frequency responses with rational function approximations [6]. This methodology has captured increasing interest in transient electromagnetic analysis and its fitting model may be easily represented by equivalent circuit with passive elements [7].

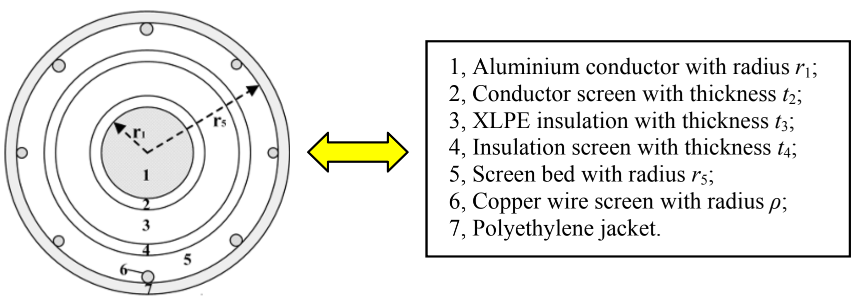


Fig.1. Geometry of XLPE cable.

* Manuscript received June 13, 2012; revised August 14, 2012.
Corresponding author. Tel.: +86-29-8266 4123; fax: +86-29-8266 8626; *E-mail address*: hjli@mail.xjtu.edu.cn.

This paper introduces the VF fitting model into the time domain analysis of single-core power cable models considering the influence of both skin effect and semi-conducting screens. Through which, the generic π circuit section of the power cable is proposed. Using the data from [4] and [5], high frequency pulse propagation is simulated by state-space method.

2. Power Cable Model

The single-core XLPE cable to be studied is shown in Fig.1, and its equivalent circuit model per unit length (p.u.l.) consist of series impedance $Z(\omega)$ and shunt admittance $Y(\omega)$ are given as follows [3],[4].

2.1 Series impedance Z(ω)

Considering the skin effect in the conductor and metallic screen, the series impedance $Z(\omega)$ is expressed by:

$$Z(\omega)=\frac{1}{2\pi r_1}\sqrt{\frac{j\omega\mu_0}{\sigma_1}}+\frac{j\omega\mu_0}{2\pi}\ln\frac{r_5}{r_1}+\frac{1}{2\pi\rho n}\sqrt{\frac{j\omega\mu_0}{\sigma_6}} \tag{1}$$

where σ_1 and σ_6 are the conductivities of aluminum and copper respectively, μ_0 is the permeability of free space, n is the total number of copper wires in the metallic screen.

2.1 Shunt admittance Y(ω)

The semi-conducting screens and XLPE insulation contribute to the shunt admittance $Y(\omega)$ of the extruded cable, which is given by:

$$Y(\omega)=\frac{1}{\sum_{i=2}^{5}\frac{1}{Y_i}} \tag{2}$$

and

$$Y_i=j\omega\frac{2\pi\varepsilon_0\varepsilon_i^*}{\ln\left(r_i/r_{i-1}\right)} \tag{3}$$

where ε_0 is the permittivity of free space, r_i and r_{i-1} are the outer and inner radius of layer i respectively, and $\varepsilon_i^*=\varepsilon_i'-j\varepsilon_i''$ is the complex permittivity for layer i, and for the XLPE insulation: ε' =2.3 and ε'' =0.001.

In addition, the complex permittivity of the semi-conducting screens obtained through measurements is described in [4], which is represented by two Cole-Cole functions and some attached terms, whereas the complex permittivity of the dispersive layers is described by Debye model in [5]. No matter what form of the complex permittivity in the cable model is, it is hard to analysis in time domain.

3. Fitting Model and Equivalent Circuits

Assume that function $F(\omega)$ represents the frequency response of a frequency dependent system, which can be rational approximated in the following form:

$$F_{fit}(\omega)=\sum_{i=1}^{N}\frac{c_i}{j\omega-a_i}+j\omega h+d=j\omega\left(\sum_{i=1}^{M}\frac{r_i}{j\omega-p_i}+h\right)+d \tag{4}$$

where r_i and p_i are residues and poles which may be either real or conjugate complex numbers, *h* and *d* are real. All the coefficients in equation (4) can be calculated by the VF procedure described in [6].

Once the fitting models are obtained, they can be easily associated with equivalent circuits with passive elements. Reference [7] shows the equivalent circuit of the p.u.l. series impedance of a lossy transmission line. Similarly, we can obtain the equivalent circuit of the p.u.l. shunt admittance of the semi-conducting layers. To facilitate the analysis, the total admittance $Y_{sc}(\omega)$ of the semiconductor layers is modeled. $Y_{sc}(\omega)$ has the same form as (3) not including the XLPE part.

Applying the VF method for $Z(\omega)$ and $Y_{sc}(\omega)$, they will be fitted in the form of (4), and their corresponding equivalent models are shown in Fig.2. In which, Fig.2(a) represents the equivalent model of $Z(\omega)$ and $R_0 = d$, $L_0 = h$, $R_i = r_i$, $L_i = -r_i/p_i$, $(i = 1 - N)$, while admittance $Y_{sc}(\omega)$ is represented in Fig.2(b) and $G_0 = d$, $C_0 = h$, $G_i = r_i$, $(i = 1 - M)$. Moreover, it should be noted that R_0 and G_0 are the corresponding parameters in the DC conditions (ω=0) for $Z(\omega)$ and $Y_{sc}(\omega)$, respectively.

4. State Space Model of Power Cables

Based on the equivalent circuit in Fig.2, the generic *k*-th π circuit section of power cable is represented in Fig.3. In which, i_{k0} and $i_{(k+1)0}$ are currents flowing through $Z(\omega)$ in the *k*-th and (*k*+1)-th π circuits, whereas v_{k0} and $v_{(k+1)0}$ are the voltage across $Y(\omega)$; G_0 and C_0 are the conductance and capacitance of the XLPE insulation, respectively.

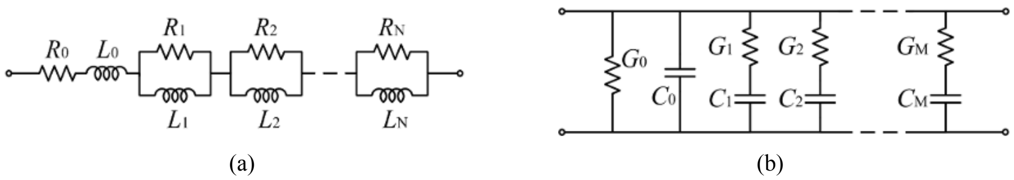


Fig.2. Equivalent circuits of fitting model for power cable. (a) Series impedance; (b) Admittance of semi-conducting layers.

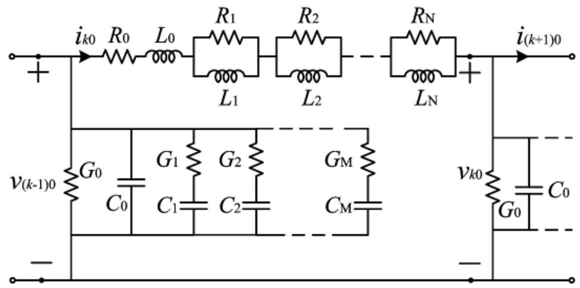


Fig.3. Equivalent π circuit for power cable.

Assume that i_{k0}, i_{k1}, i_{k2}, …, i_{kN} are currents through L_0, L_1, L_2, …, L_N, and v_{k1}, v_{k2}, …, v_{kM} are voltages across C_1, C_2, …, C_M, respectively. Considering *K*π sections connected in cascade, the state differential equations in time domain can be written as:

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{B}v_s \tag{5}$$

where $\mathbf{X}$ and $\mathbf{B}$ are vectors with $(M+N+2)K$ dimensions, and $\mathbf{A}$ is $(M+N+2)K \times (M+N+2)K$ dimensional matrix, v_s is excitation source, their specific forms are as follows:

$$\mathbf{X}^{\mathrm{T}} = \left[\mathbf{X}_1\ \mathbf{X}_2 \cdots \mathbf{X}_K\right] \tag{6}$$

$$\mathbf{B}^{\mathrm{T}} = \left[1/L_0\ \ 0\ \ 0 \cdots 0\right] \tag{7}$$

$$\mathbf{A}=\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & & & \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} & & \\ & \ddots & \ddots & \ddots & \\ & & \mathbf{A}_{(K-1)(K-2)} & \mathbf{A}_{(K-1)(K-1)} & \mathbf{A}_{(K-1)K} \\ & & & \mathbf{A}_{K(K-1)} & \mathbf{A}_{KK} \end{bmatrix} \tag{8}$$

And for the k-th π section, the related parameters are:

$$\mathbf{X}_k^{\mathrm{T}}=\left[i_{k0}\ i_{k1}\ \ i_{k2}\cdots\ i_{kN}\ v_{k0}\ v_{k1}\ v_{k2}\cdots v_{kM}\right] \tag{9}$$

The generic submatrix $\mathbf{A}_{kk}$ with $(\mathrm{M}+\mathrm{N}+2)$ dimensions is expressed by:

$$\mathbf{A}_{kk}=\begin{bmatrix} -\frac{\sum_{i=0}^{N}R_i}{L_0} & \frac{R_1}{L_0} & \frac{R_2}{L_0} & \cdots & \frac{R_N}{L_0} & -\frac{1}{L_0} & 0 & 0 & \cdots & 0 \\ \frac{R_1}{L_1} & -\frac{R_1}{L_1} & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ \frac{R_2}{L_2} & 0 & -\frac{R_2}{L_2} & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{R_N}{L_N} & 0 & 0 & \cdots & -\frac{R_N}{L_N} & 0 & 0 & 0 & \cdots & 0 \\ \frac{1}{C_0}+\frac{1}{C_1} & 0 & 0 & \cdots & 0 & -\frac{G_0}{C_0} & \frac{G_0}{C_0}-\frac{\sum_{i=1}^{M}G_i}{C_1} & \frac{G_1}{C_1} & \cdots & \frac{G_M}{C_1} \\ \frac{1}{C_1} & 0 & 0 & \cdots & 0 & 0 & -\frac{\sum_{i=1}^{M}G_i}{C_1} & \frac{G_2}{C_1} & \cdots & \frac{G_{\mathrm{M}}}{C_1} \\ 0 & 0 & 0 & \cdots & 0 & 0 & \frac{G_2}{C_2} & -\frac{G_2}{C_2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \frac{G_M}{C_M} & 0 & \cdots & -\frac{G_M}{C_M} \end{bmatrix} \tag{10}$$

$\mathbf{A}_{k(k\text{-}1)}$ and $\mathbf{A}_{(k\text{-}1)k}$ having the same dimensions as $\mathbf{A}_{kk}$ are very highly sparse, and there are only a few non-zero elements in each of them, where:

$$\begin{cases} \mathbf{A}_{\mathrm{k(k\text{-}1)}}(1,(N+2)K)=\dfrac{1}{L_0} \\ \mathbf{A}_{\mathrm{(k\text{-}1)k}}((N+2)K,1)=-(\dfrac{1}{C_0}+\dfrac{1}{C_1}), \quad \mathbf{A}_{\mathrm{(k\text{-}1)k}}((N+2)K,1)=-\dfrac{1}{C_1} \end{cases} \tag{11}$$

The state equations (5) are first-order linear ordinary differential equations and they can be easily solved with numerical methods. In this paper, Runge-Kutta method was used to solve the state space model.

5. Calculation Examples and Discussion

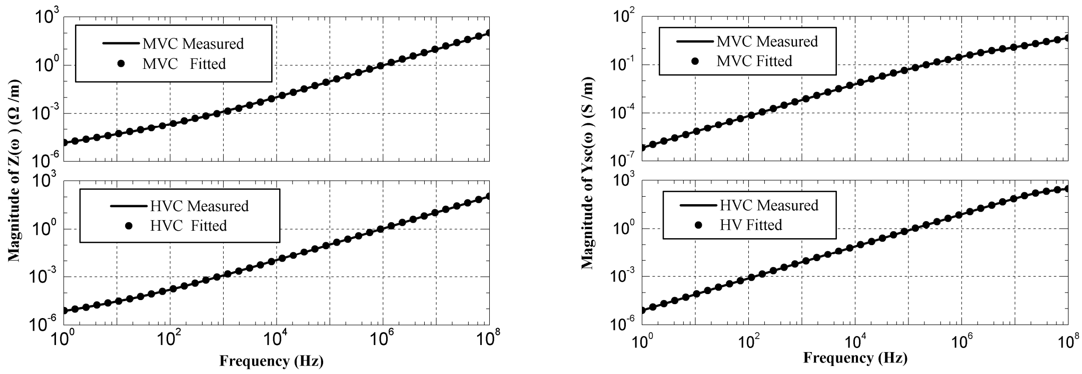
This section presents example for the purpose of applying the proposed model. The XLPE cables to be studied are from the medium voltage cable called cable 1 in [4] (MVC) and the high voltage cable in [5] (HVC). The complex permittivity forms of which are Cole-Cole and Debye models, respectively. The details of the cables investigated are in Table 1.

Table 1. Cable parameters used in calculation (All units are in mm).

Parameter	MVC	HVC
Conductor radius (r_1)	7.3	15
Conductor screen thickness (t_2)	0.6	1.3
Insulation thickness (t_3)	5.5	18
Insulation screen thickness (t_4)	0.4	1.3
Screen bed radius (r_5)	15.8	35.6

5.1. Modelling results by vector fitting

In the rational approximation of the series impedance $Z(\omega)$ and the total admittance $Y_{sc}(\omega)$ of the semiconductor layers for both MVC and HVC, vector fitting procedure was implemented in the range of 0.01Hz to 100MHz. Fig.4 shows the fitting results after 20 iterations using 2 pairs of complex starting poles. It is seen that very good approximations have been achieved. The maximum of the root-mean-square (RMS) errors was found to be 1.88E-8.



(a) (b)

Fig.4. Vector Fitting results of MVC and HVC. (a) Magnitude of $Z(\omega)$ p.u.l.; (b) Magnitude of $Y_{sc}(\omega)$ p.u.l..

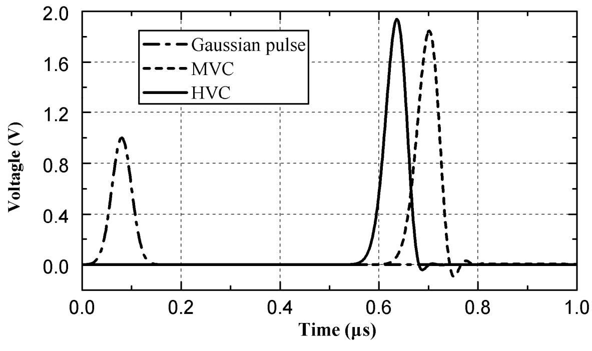


Fig.5. Equivalent π circuit for power cable.

Table 2. Fitting Parameters of Series Impedance Shown in Fig.3.

Resistance (Ω/m)			Inductance (H/m)		
Parameter	MVC	HVC	Parameter	MVC	HVC
R0	9.7332×10^{-4}	3.9298×10^{-5}	L0	1.6261×10^{-7}	1.7527×10^{-7}
R1	3.4701×10^{-4}	2.9315×10^{-4}	L1	1.2757×10^{-7}	3.7109×10^{-8}
R2	5.0486×10^{-5}	4.2661×10^{-5}	L2	3.2066×10^{-7}	1.0949×10^{-7}
R3	1.4774×10^{-5}	1.1480×10^{-5}	L3	7.5397×10^{-7}	3.1659×10^{-7}
R4	8.9996×10^{-6}	5.3953×10^{-6}	L4	4.7701×10^{-6}	1.9469×10^{-6}

Table 3. Fitting Parameters of Shunt Admittance Shown in Fig.3

Conductance (S/m)			Capacitance (F/m)		
Parameter	MVC	HVC	Parameter	MVC	HVC
G0	1.0514×10^{-13}	7.4673×10^{-13}	C0	2.4182×10^{-10}	1.7175×10^{-10}
G1	1.1599×10^{2}	5.2223×10^{1}	C1	4.8327×10^{-9}	2.3088×10^{-7}
G2	5.9236×10^{-9}	4.0945×10^{-21}	C2	1.9717×10^{0}	1.3118×10^{-10}
G3	2.5524×10^{-8}	1.0159×10^{-6}	C3	7.0381×10^{-1}	2.2574×10^{2}
G4	3.5475×10^{-8}	3.6614×10^{-22}	C4	1.3123×10^{-1}	1.0983×10^{-14}
G5	3.1862×10^{-8}	3.1824×10^{-23}	C5	1.1743×10^{-2}	2.9021×10^{-20}

Also, the frequency dependent parameters in Fig.3 (M=N=4) are given in Table 2 and Table 3 in Appendix.

5.2. Transient simulation

In the following calculations, the length of the extruded power cables is 100 meters. Determining the number of π sections is important in the transient analysis of system represented by lumped-parameter π circuits connected in cascades. In this paper, the statistical correlation method was adopted which was implemented by the following procedure.

Step 1) Choose K_0 as the initial number of π sections.

Step 2) Calculate the voltages V_1 and V_2 at the observation points with K_0 and $K_0+\Delta K$ π sections considered, respectively.

Step 3) Compute the Pearson correlation coefficient C_0 of V_1 and V_2. If C_0 is greater than a setting value, accept K_0 as the final result; otherwise set $K_0 = K_0+\Delta K$, go to Step 2.

Based on the method above, the cable was finally divided into 100 π circuits connected in cascades. Simulation was carried out between 0 to 1 μs with 1 ns as the calculation step. Gaussian pulse with frequency width of 20MHz is injected into the XLPE cables, and the open-circuit voltages at the receiving end are shown in Fig.5. From which we can obtain the propagation velocities of MVC and HVC are about 162m/μs and 179 m/μs, respectively. Furthermore, there is higher attenuation in MVC for the same injected pulse.

6. Conclusion

The characteristics of frequency dependent parameters in power cables can be efficiently rational approximated by VF method. Based on the equivalent circuits proposed in this paper, it is easy to obtain the state space model in time domain for extruded cable models considering skin effects and semi-conducting screens.

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